An introduction to the methodologies for studying complex networks

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Advanced Field School in Computational Ecology 2023

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1. What is Network Science? Where does it come from?

2. How to use Network Science?

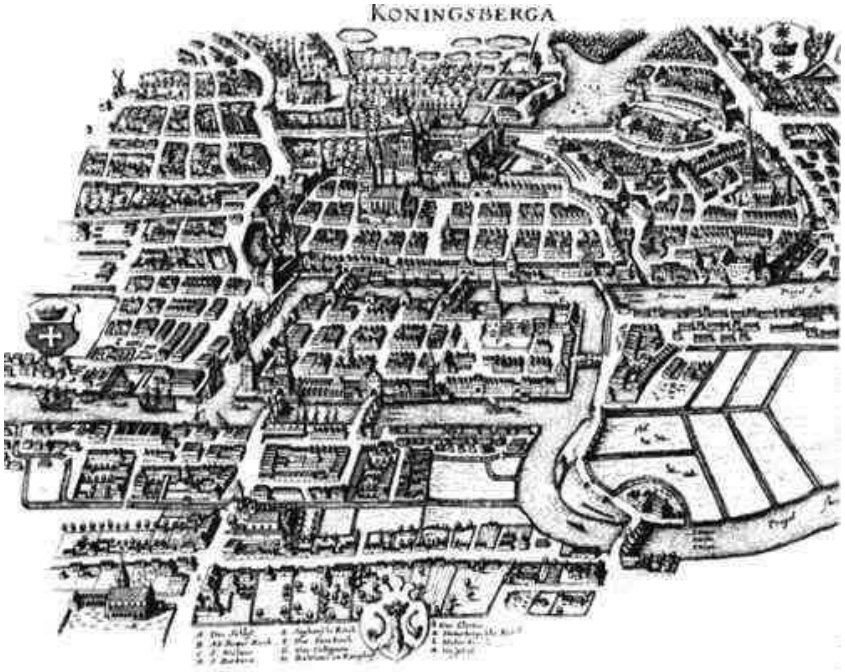
3. How to learn more about Network Science?

What is Network Science? Where does it come from?

The Seven Bridges of Königsberg

Contemporary puzzle (c. 1735):

Can one walk across all seven bridges and never cross the same one twice?



en.wikipedia.org/wiki/Seven_Bridges_of_Königsberg

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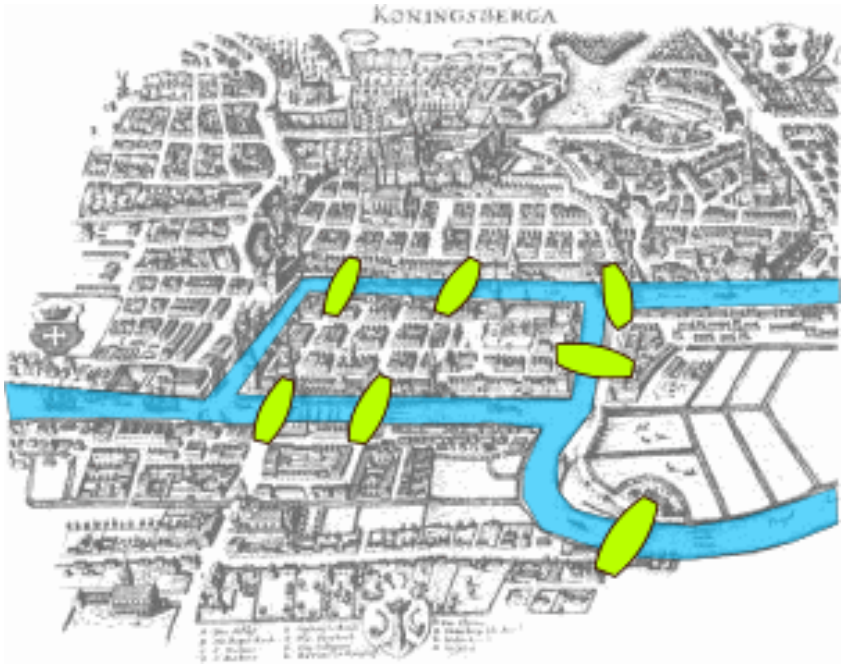

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The Seven Bridges of Königsberg

Contemporary puzzle (c. 1735):

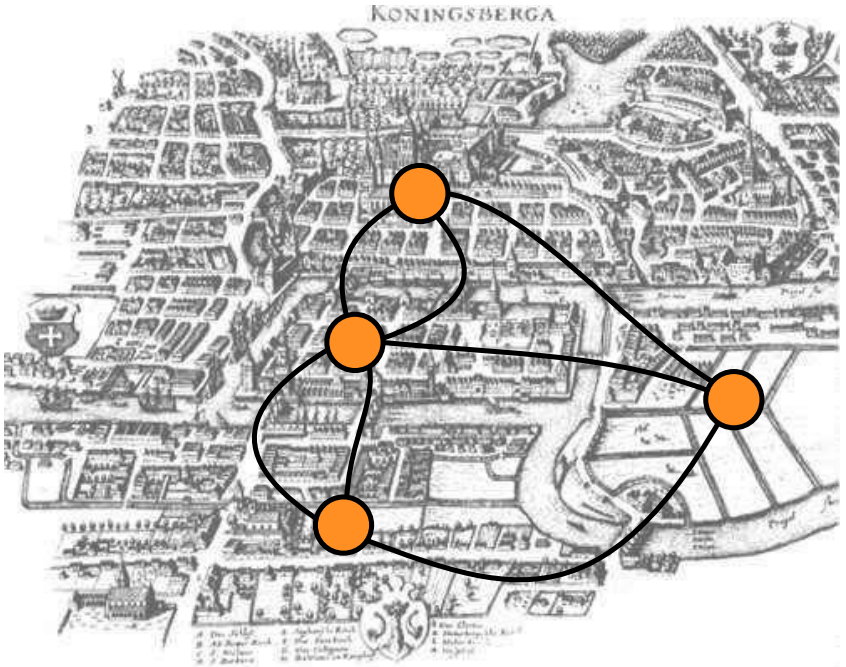
Can one walk across all seven bridges and never cross the same one twice?

Euler's idea:

Get rid of superfluous information to focus on the ***structure*** *of the problem.*

Conclusion:

Such a path cannot exist on a graph that has ***more than two nodes*** *with an* ***odd number of connections (degree)****.*



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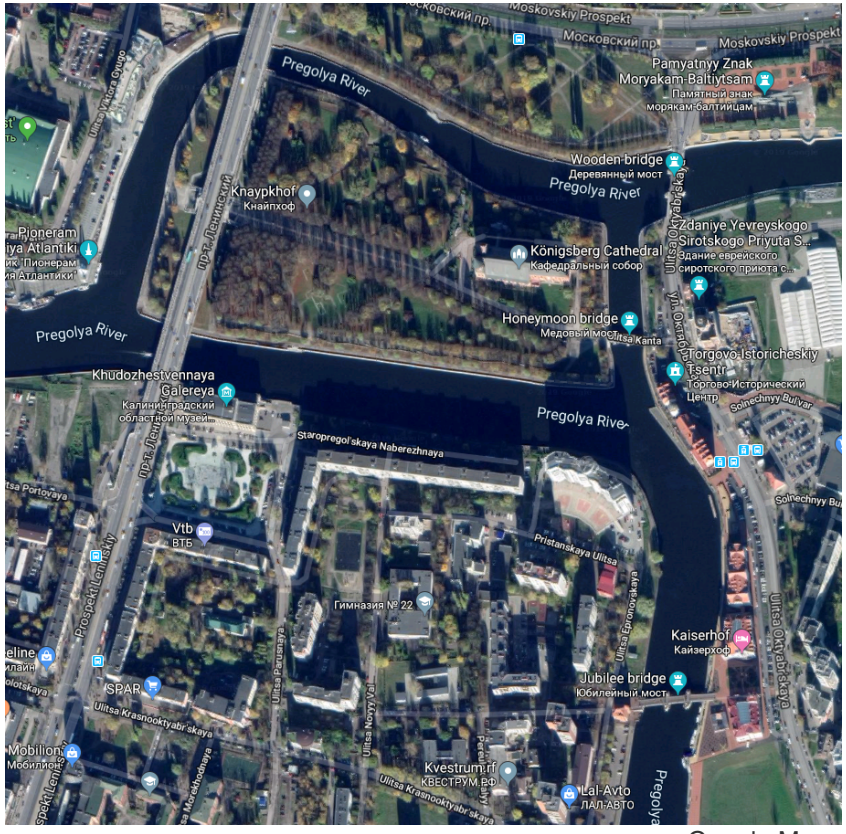
The Seven Bridges of Königsberg

Contemporary puzzle (c. 2023):

- Only two of the original bridges remain.
- Two did not survive the bombing of Königsberg in World War II.
- Two others were later demolished and replaced by a modern highway.
- One was rebuilt elsewhere in 1935.

*Can one walk across all **five** bridges and never cross the same one twice?*

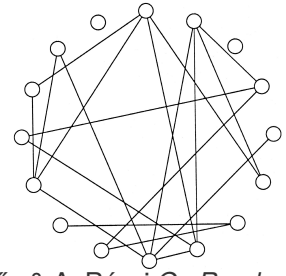


Google Maps

Graph Theory and Sociology

Euler's idea is at the origin of Graph Theory

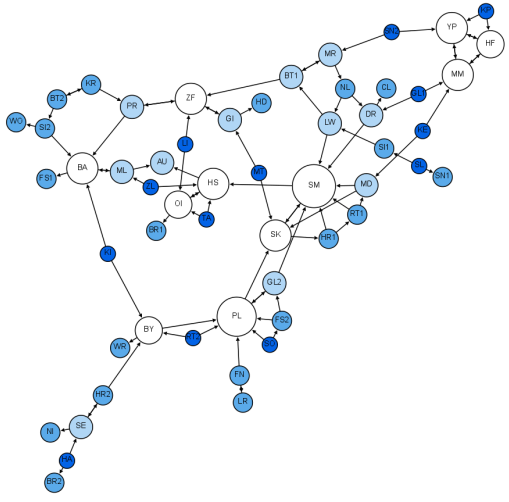
Ex. random graphs



P. Erdős & A. Rényi *On Random Graphs I* (1959)
PNAS 99:2566 (2002)

Meanwhile, in Sociology…

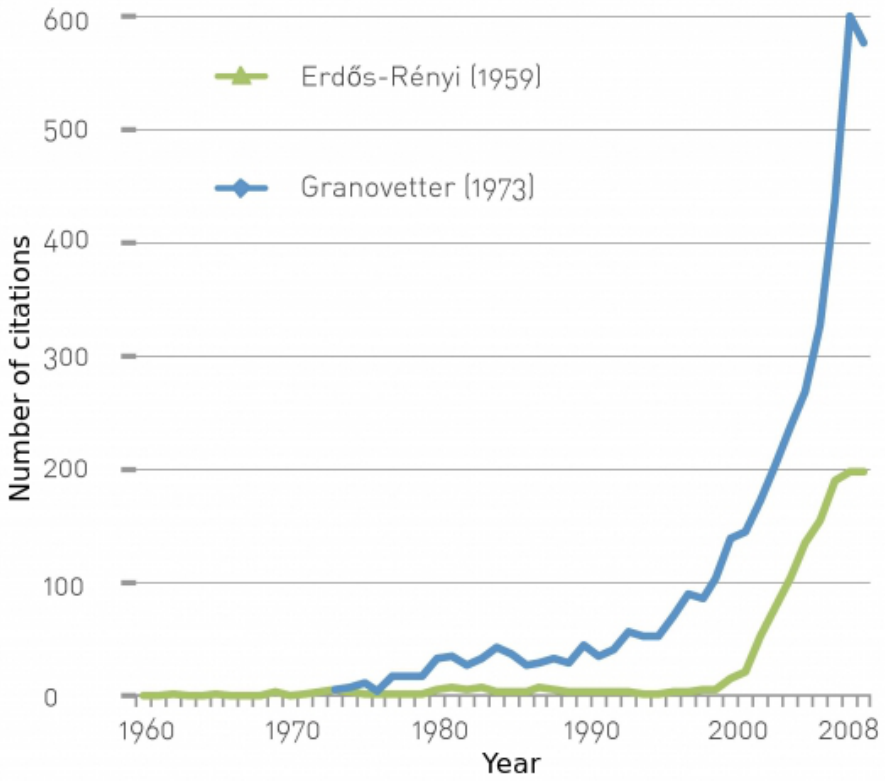
Ex. sociograms



J. L. Moreno *Who shall survive?* (1934)

Graph Theory and Sociology

Modest impact outside Mathematics and Sociology until the late 1990s



A.-L. Barabási *Network Science* (2016)

Emergence of a “new” discipline

The emergence of Network Science has been driven by two forces.

1) The digital revolution made possible the extraction of the **wiring diagram** of many complex systems.

Emergence of a “new” discipline

What makes a **system complex**?

→ **large number** of elements

→ non-trivial **interactions**

→ **emergent** behavior(s)

“the whole is more than the sum of the parts”

“beyond the grasp of the reductionist approach”

Examples:

- ✓ ecosystems
- ✓ living organisms
- ✓ brains
- ✓ on-/off-line social networks
- ✓ Internet
- ✓ economies
- ✓ power grids
- ✗ cars

Emergence of a “new” discipline

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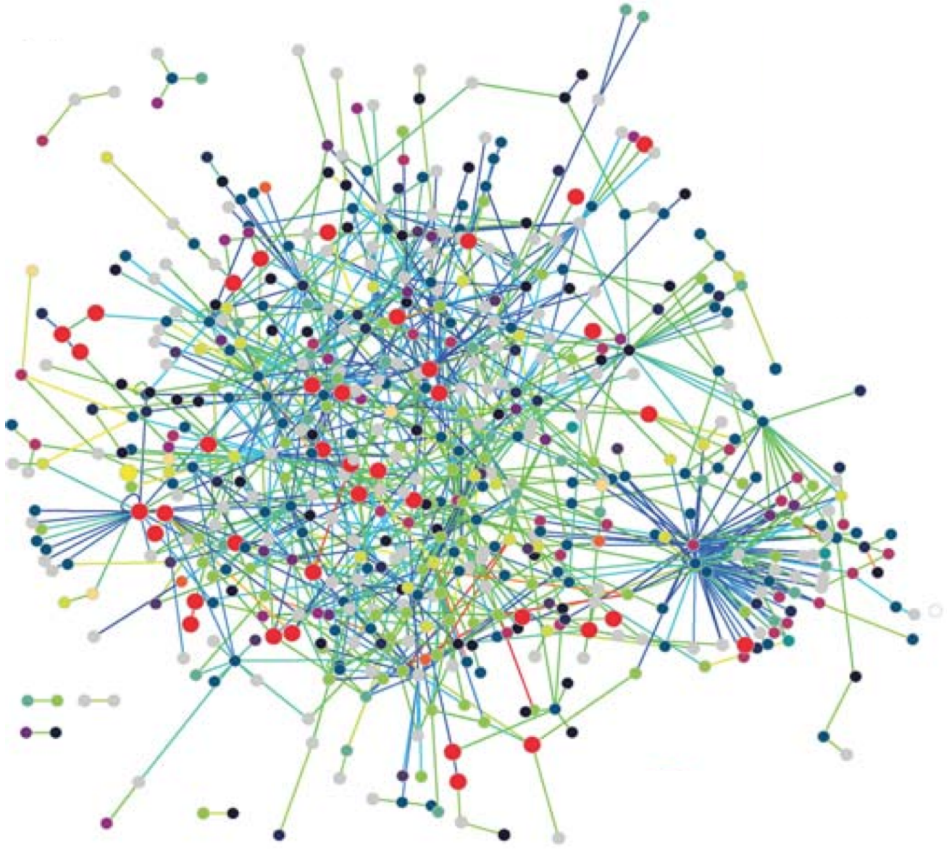
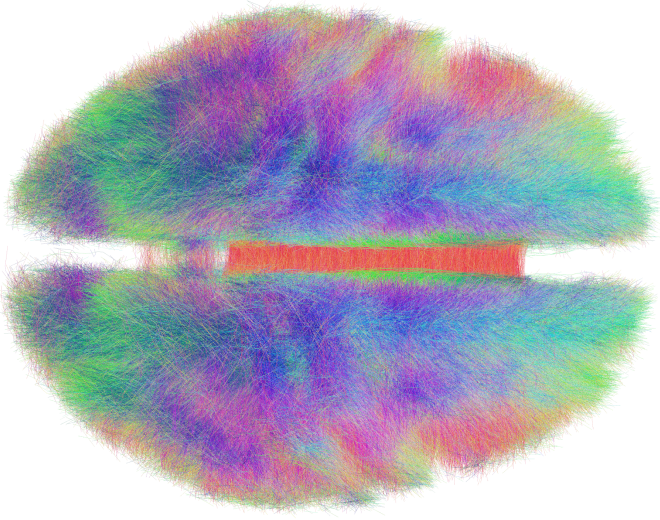


Protein interaction network of T. pallidum

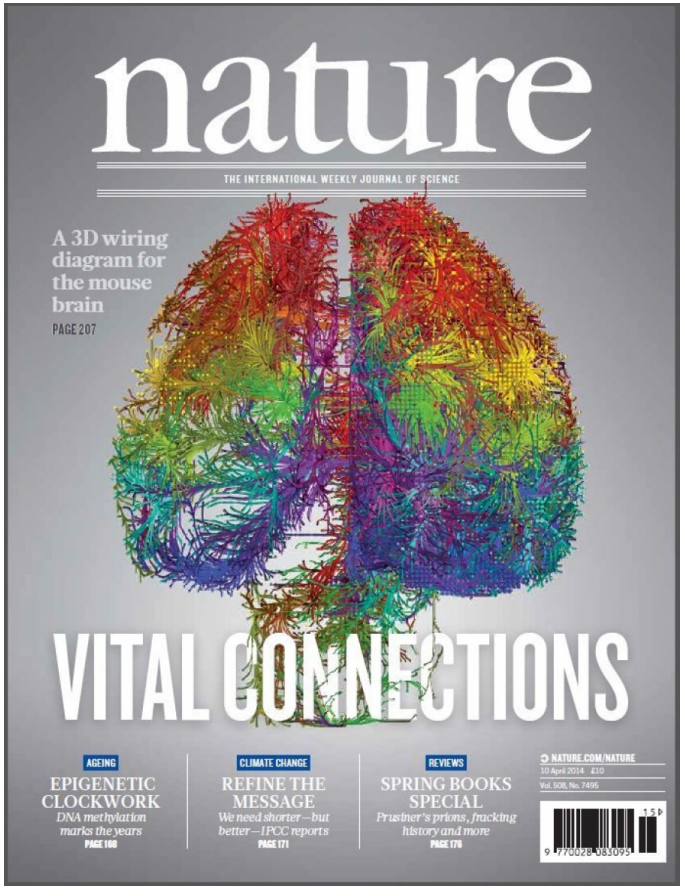
PLOS ONE 3:e2292 (2008)

Emergence of a “new” discipline



White matter architecture of the human brain

Neuroimage 102:142 (2014)



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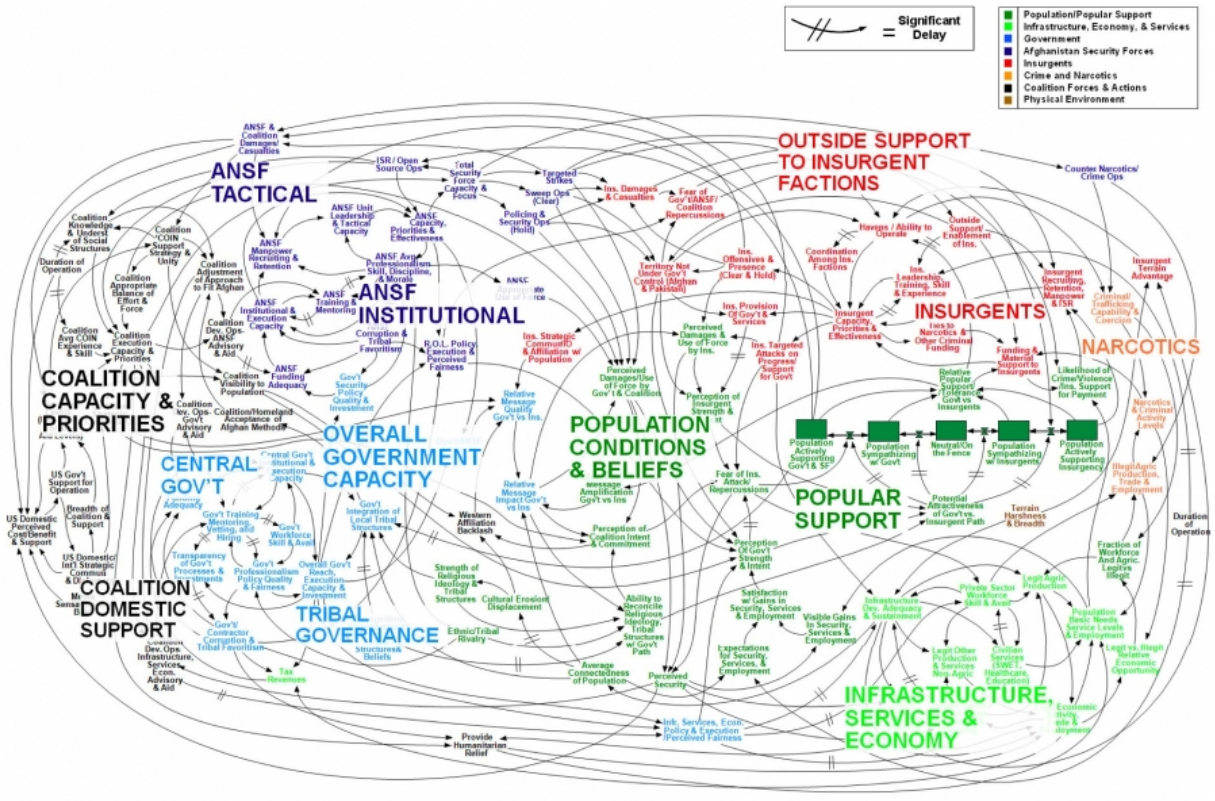
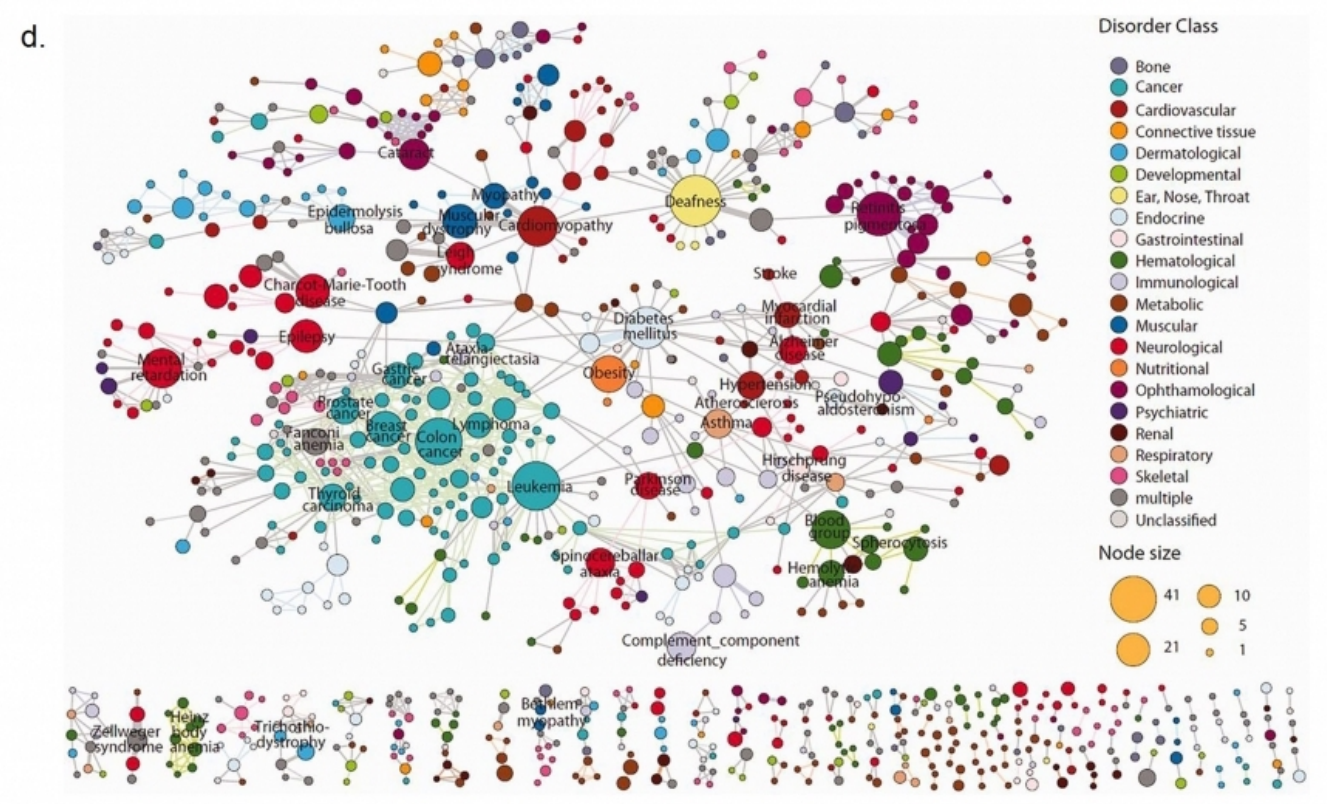


American operational plans during the Afghan war in 2012

New York Times

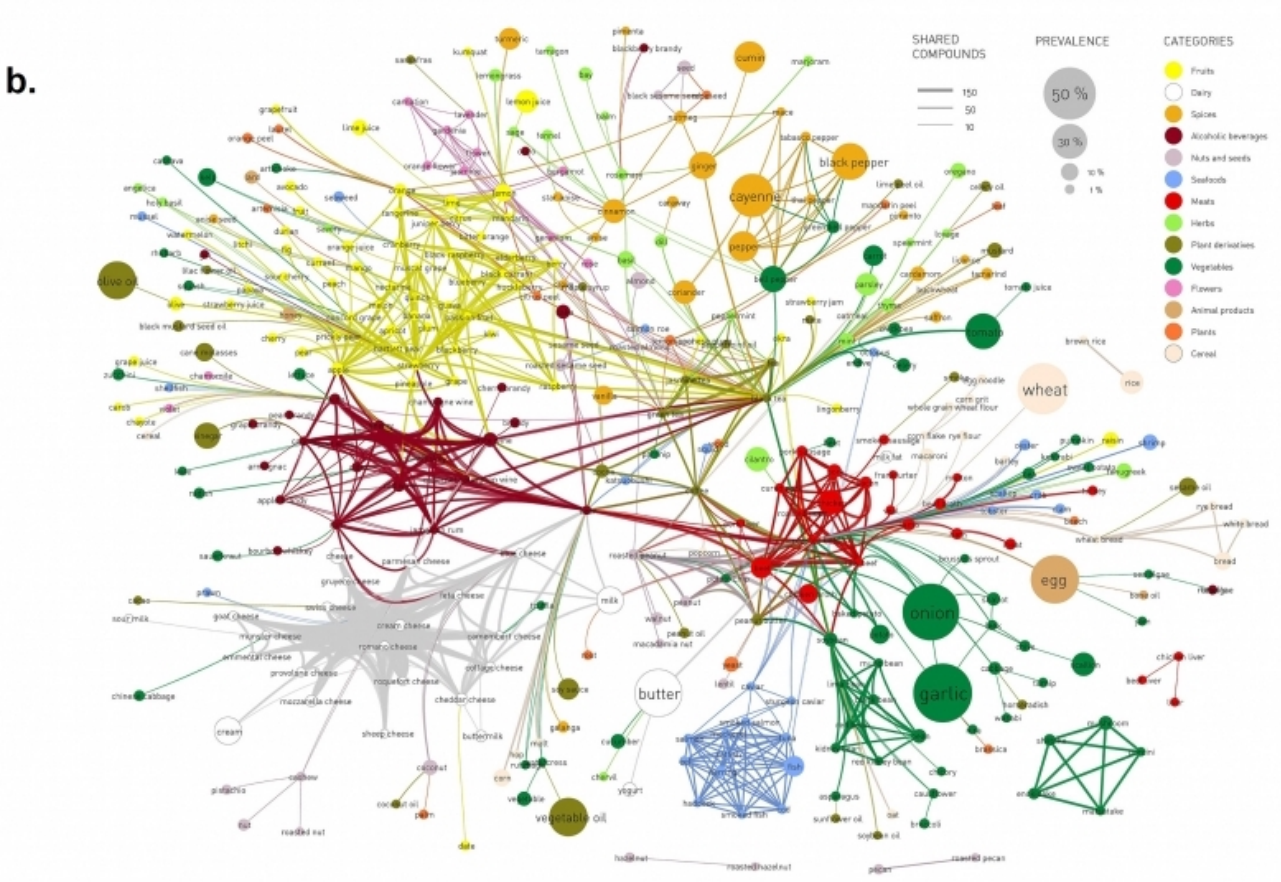
Emergence of a “new” discipline



Projection of the Human Disease Network

K.-I. Goh *et al.* PNAS, 104:8685 (2007)

Emergence of a “new” discipline



Flavor network in which ingredients are connected if they share a significant number of flavor compounds. Link thickness represents the number of shared compounds.

Y.-Y. Ahn *et al.* Sci. Rep. 196 (2011)

Emergence of a “new” discipline

Position of the leadership within the informal network of employees of a Hungarian company

A.-L. Barabási *Network Science* (2016)

Emergence of a “new” discipline

The emergence of Network Science has been driven by two forces.

1) The digital revolution made possible the extraction of the **wiring diagram** of many complex systems.

2) The **universality** of the architecture of these wiring diagrams across various domains.

Emergence of a “new” discipline

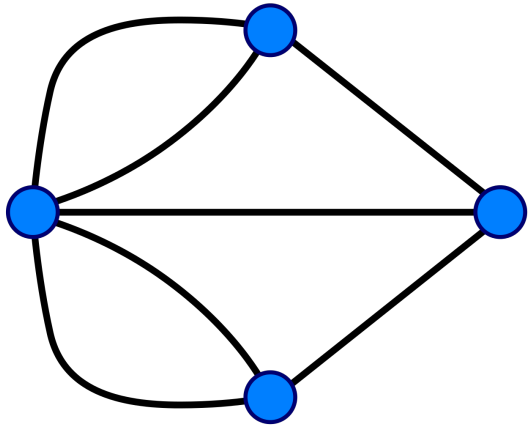
Complex networks: A unifying paradigm to study and model complex systems

→ elements = nodes

→ interactions = links

→ remove superfluous information

→ focus on the structure of the interactions



A **single** mathematical representation for complex systems of **diverse** natures

→ “**universal**” key organizational principles

→ characterized using **common measures**

★ Tools, models and concepts “can be transferred” to other contexts

How to use Network Science?

How to use Network Science

1) Choose a network representation of the dataset

- What are the nodes (i.e., interacting elements)?
  - Ex.: species, metabolites, proteins, genes, neurons, cities, airports, railway stations, network routers, ISP, email addresses, mobile phone numbers, articles, webpages, people
  - Are all nodes equal? Are there categories?
- What are the links (i.e., interactions)?
  - Ex.: predation, chemical reactions, binding, regulation, synapses, roads, flights, packets, messages, calls, citations, hyperlinks, friendship, collaborations, sexual contacts, authority, gossip
  - Are they unidirectional/bidirectional?
  - Are they binary/weighted?
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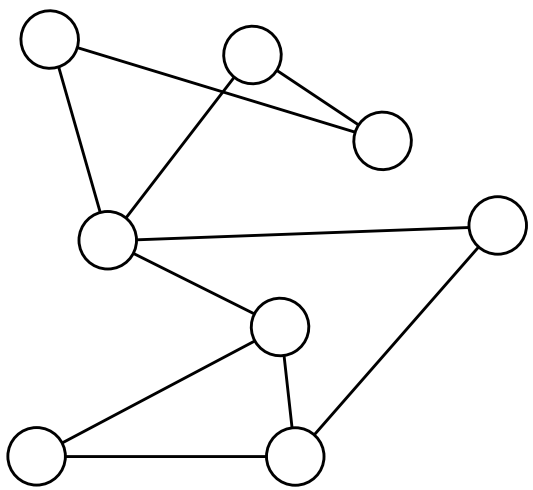

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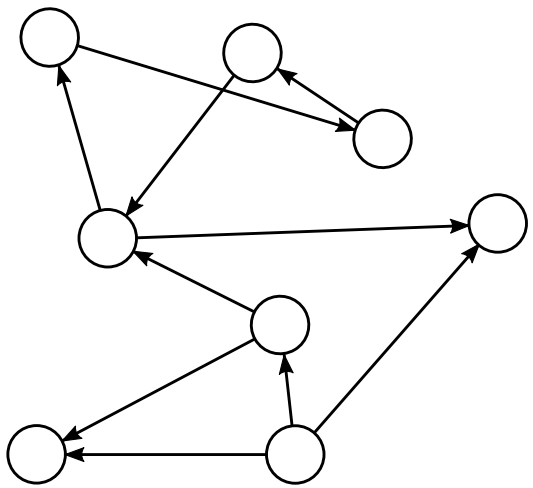

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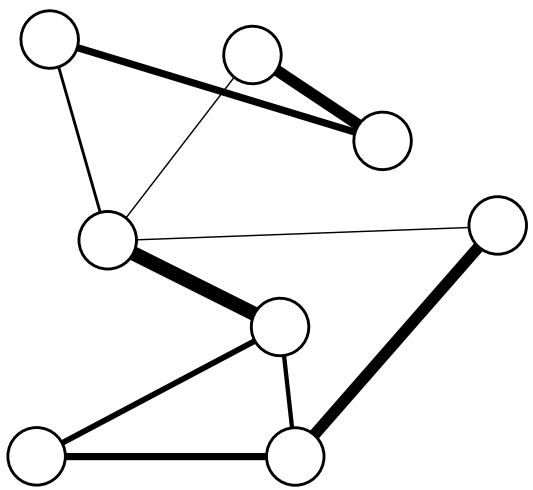

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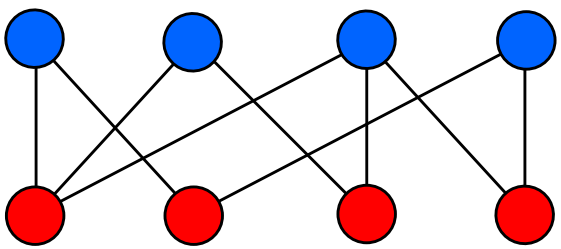

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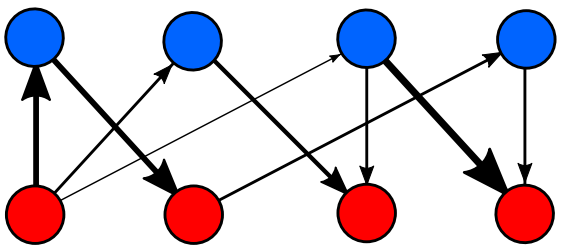

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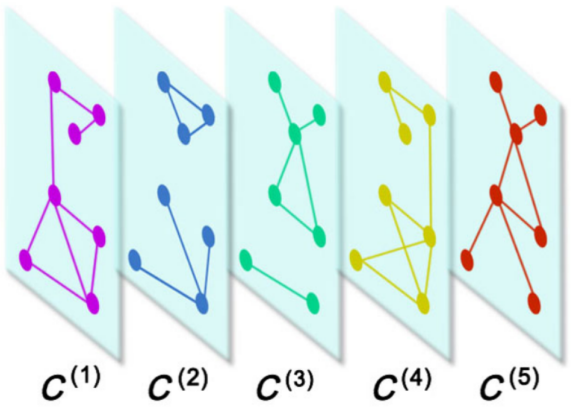

Phys. Rev. X 3:041022 (2013)

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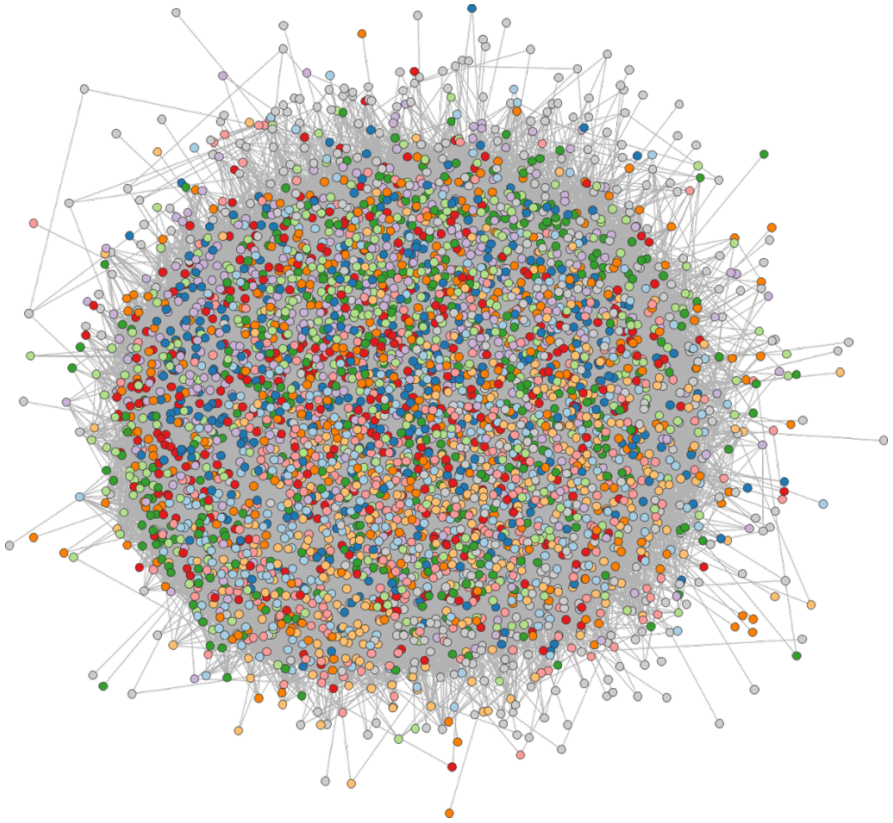
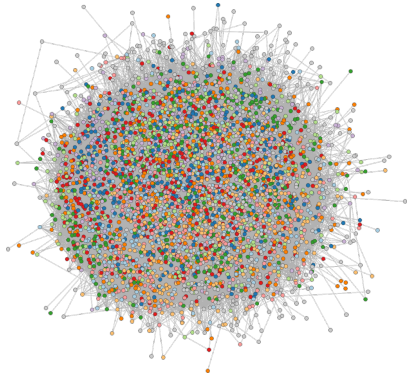


visual-computing.org

How to use Network Science?

Many unanswered questions:

- How are the links organized?
- Are some nodes more *important* than others?
- Are there underlying surprising patterns?
- Can the organization of the network be explained some growth processes?
- What does the micro/meso/macroscale organization look like?
- …



How to use Network Science?

2) Five general approaches to “disentangle the hairball”

1. **Exploratory data analysis**: measure and compare various basic quantities (degree, centrality scores, correlations, etc.)

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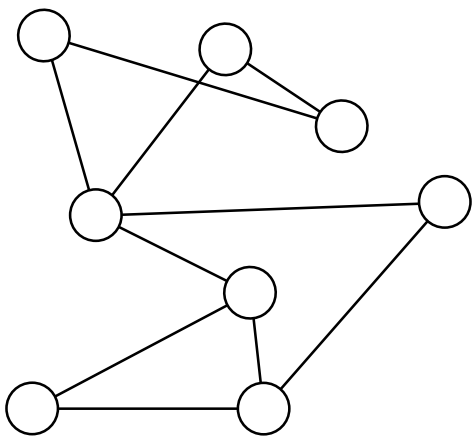

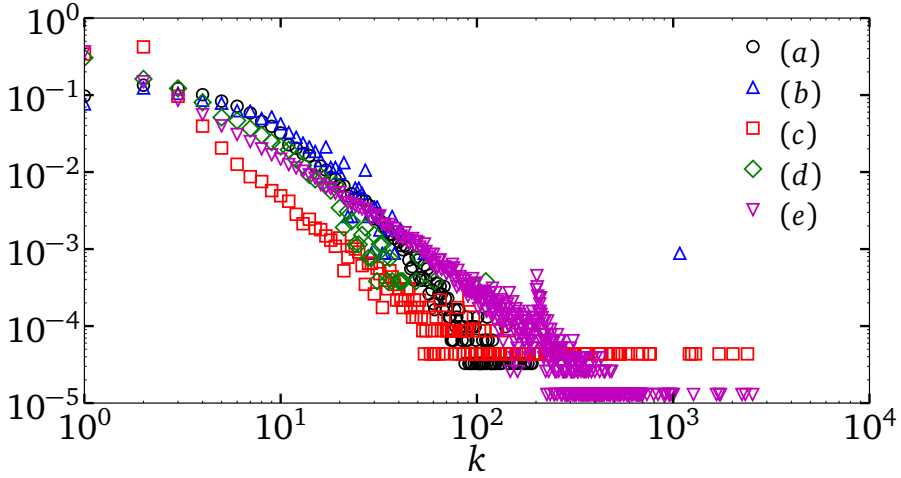

High-degree nodes (hubs) are critical in keeping the network connected, and govern many spreading processes.

How to use Network Science?

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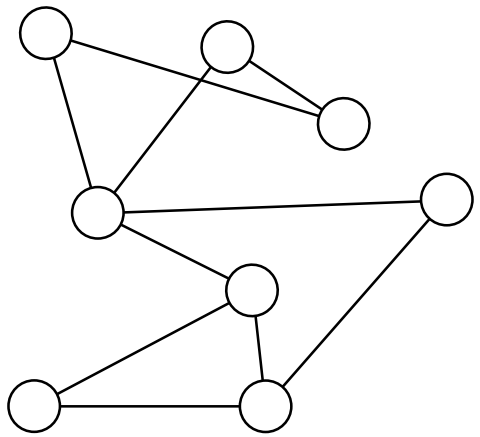

Many, many ways to measure centrality:

- closeness centrality
- harmonic centrality
- betweenness centrality
- eigenvalue centrality
- PageRank
- k-core decomposition
- onion decomposition
- ...

How to use Network Science?

2) Five general approaches to “disentangle the hairball”

1. **Exploratory data analysis**: measure and compare various basic quantities (degree, centrality scores, correlations, etc.)
2. **Null models**: use some kind of random network model to identify non-random patterns as deviations from the null hypothesis (community structure, etc.)

How to use Network Science?

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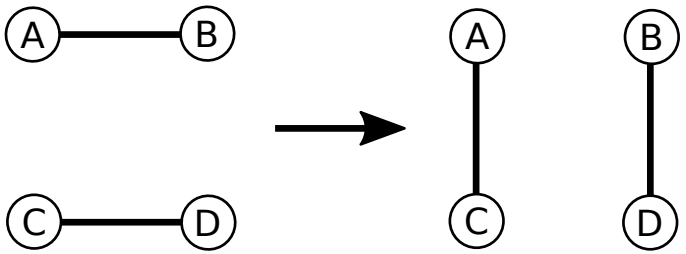
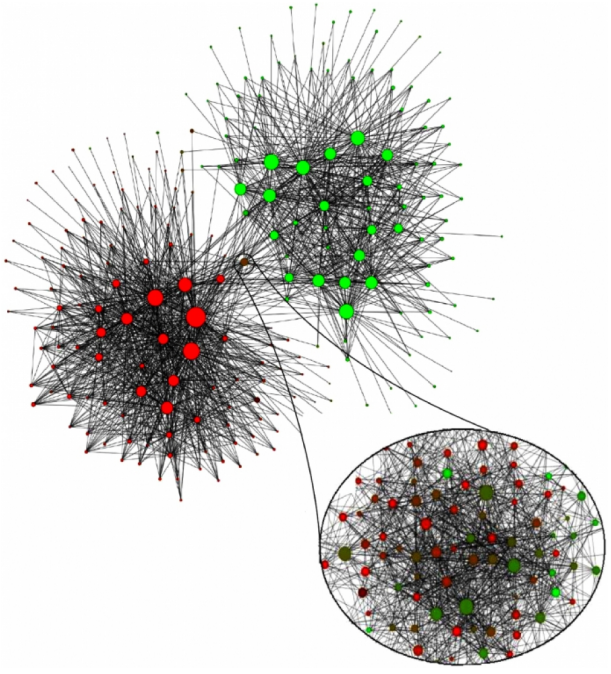
Choose two pairs of links at random and swap them.





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2. **Null models**: use some kind of random network model to identify non-random patterns as deviations from the null hypothesis (community structure, etc.)
3. **Mechanisms/simulations**: explain structural or dynamical patterns as caused by specific process
4. **Predictive models**: fit parametric model of network structure, and use it to predict missing or future data (edges, labels, etc.)
5. **Network experiments**: manipulate structure and measure node-level or network-level behavior as function of changes.

How to use Network Science?

Many packages exist to facilitate the analysis/visualization of networks:

- NetworkX [python]
- graph-tool [python, c++]
- iGraph [python, c++, R]
- GraphLab [python, c++]
- BaseGraph [python, c++]
- TACOMA: TemporAl COntact Modeling and Analysis [python]
- XGI: CompleX Group Interactions [python]
- Gephi
- Pajek
- Cytoscape
- Graphviz
- ...

How to learn more about Network Science?

How to learn more about Network Science?

Very good textbooks published in recent years

Online resources:

- `networksciencebook.com`
- `https://www.networkatlas.eu/`
- `mrpandey.github.io/d3graphTheory`

How to learn more about Network Science?

CNWW2023 in December?

Follow @CNWWs to stay tuned!

NetSci

The flagship conference on Network Science will be held in Québec City in June 2024.