Modeling Complex and Interacting Contagions with Approximate Master Equations

Antoine Allard

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Outline

1. The model and its mathematical description

2. Some applications

   (a) Mesocopic localization

   (b) Heterogeneous transmission settings

   (c) Context-sensitive behavior



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PHYSICAL REVIEW E **82**, 036115 (2010)

Propagation dynamics on networks featuring complex topologies

Laurent Hébert-Dufresne, Pierre-André Noël, Vincent Marceau, Antoine Allard, and Louis J. Dubé



Louis J. Dubé
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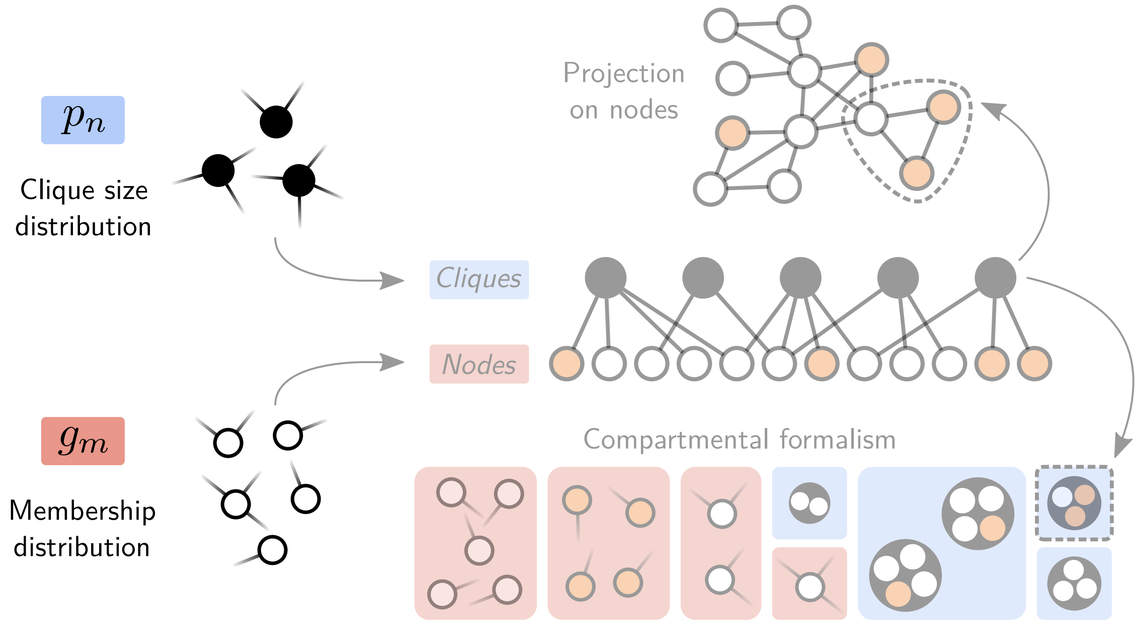


Vincent Marceau
Auth0

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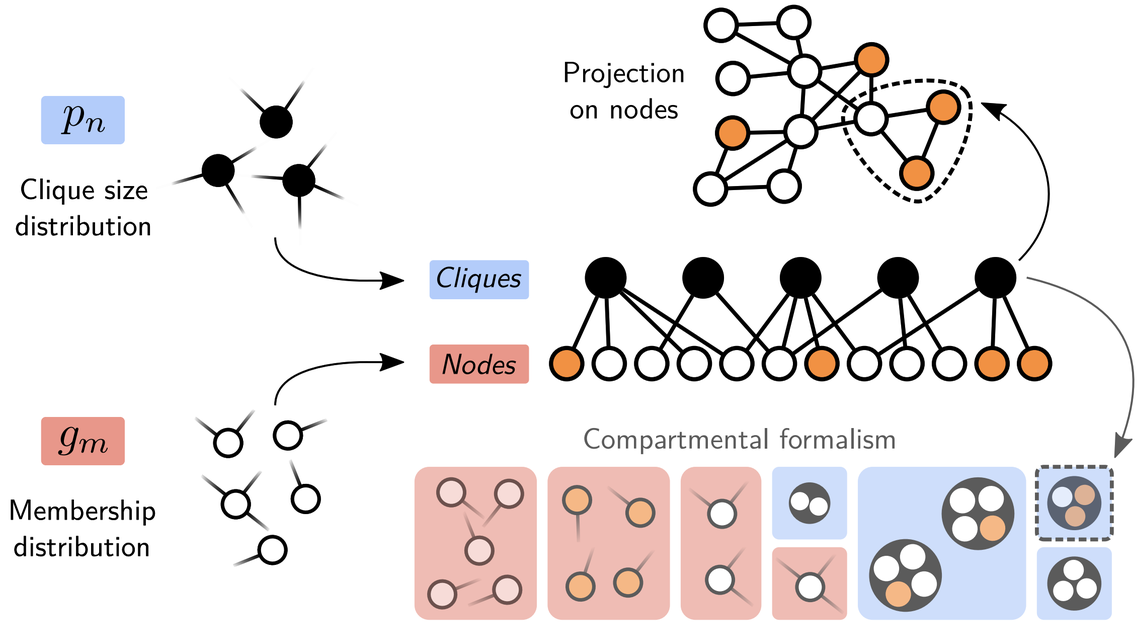
The model and its mathematical description



g_m : fraction of nodes with membership m
p_n : fraction of groups of size n

M.E.J. Newman, *Properties of highly clustered networks*, Phys. Rev. E 68, 026121 (2003)

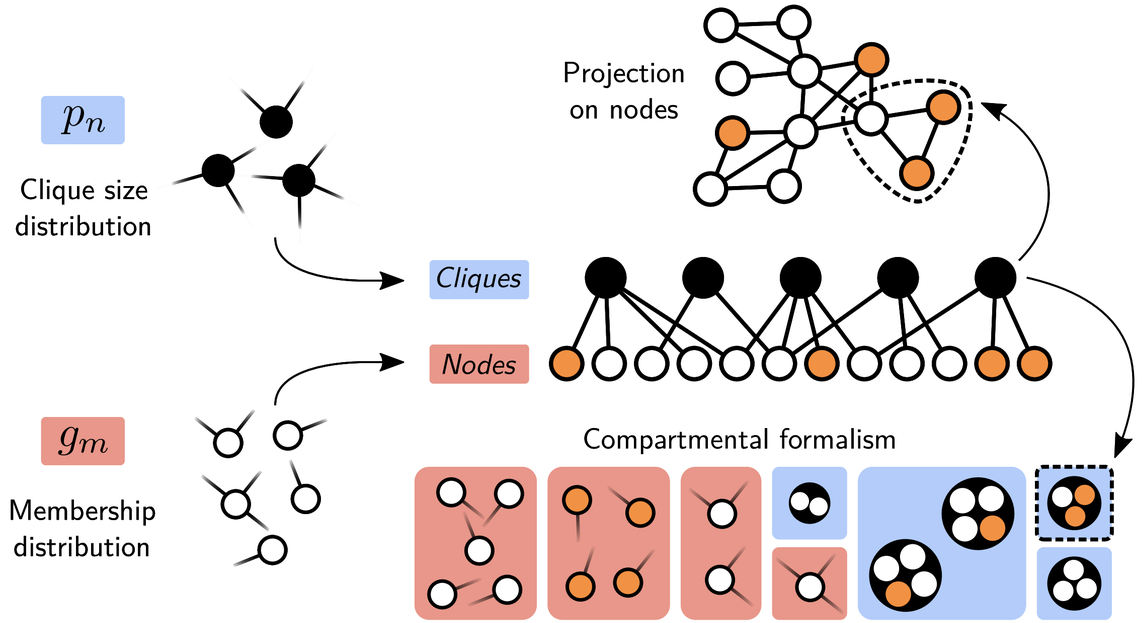
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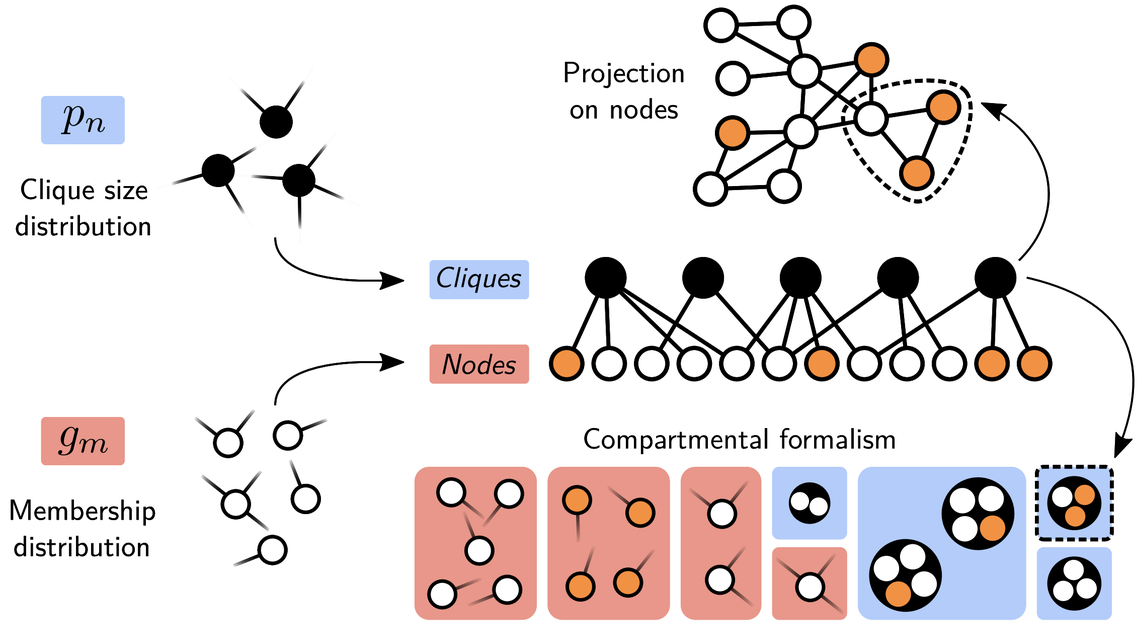


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S_m : fraction of nodes with membership m and that are susceptible
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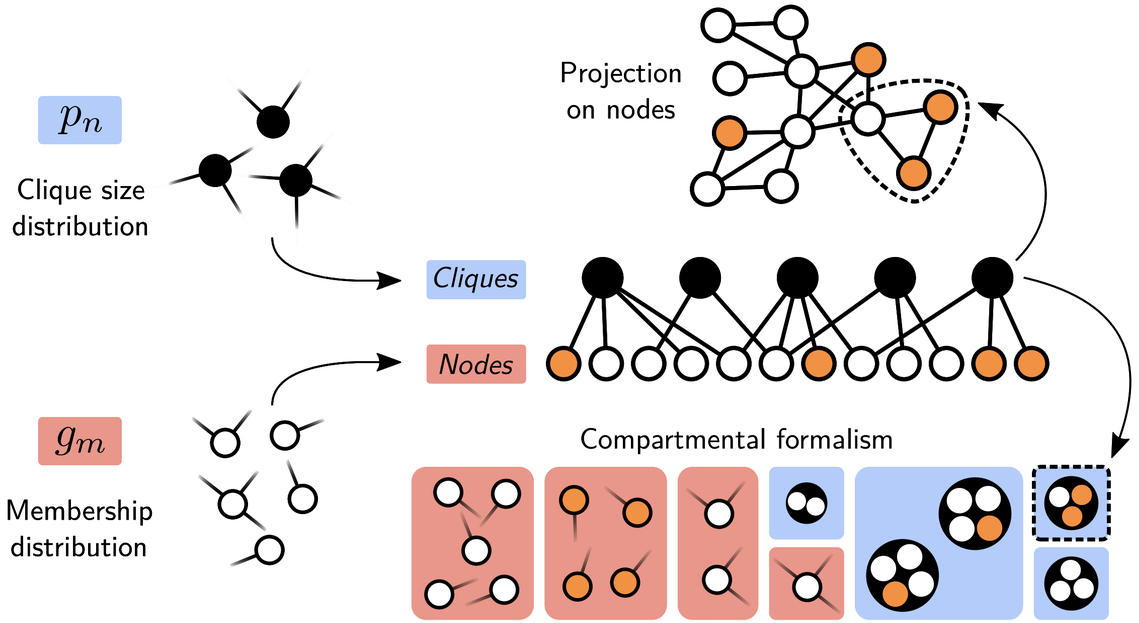
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β : *weight* associated with transmission
$\Theta_{n,i,\beta}$: transmission rate in a group of size n, with i infected nodes, and weight β

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Dynamics at the groups level

$$\begin{aligned}\frac{\mathrm{d}G_{n,i}}{\mathrm{d}t} &= \mu(i+1)G_{n,i+1} - \mu i G_{n,i} \\ &+ (n-i+1)\Theta_{n,i-1,\beta}G_{n,i-1} - (n-i)\Theta_{n,i,\beta}G_{n,i} \\ &+ (n-i+1)\rho G_{n,i-1} - (n-i)\rho G_{n,i}\end{aligned}$$

Dynamics at the nodes level

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} = \mu(g_m - S_m) - m r S_m$$

$r(t)$: mean infection rate from a random group to which a random node belongs
$\rho(t)$: mean infection rate from all *other* groups to which a random node in a random group belongs

Mean-field quantities

$$\rho(t) = r(t)\frac{\sum_m m(m-1)S_m}{\sum_m mS_m} \; ; \quad r(t) = \frac{\sum_{n,i}(n-i)\Theta_{n,i,\beta}G_{n,i}}{\sum_{n,i}(n-i)G_{n,i}}$$

Global prevalence

$$I(t) = \sum_m [g_m - S_m(t)]$$

M.E.J. Newman, *Properties of highly clustered networks*, Phys. Rev. E 68, 026121 (2003)

The model and its mathematical description

Stationary state

$$
\begin{aligned}
S_m^* &= \frac{g_m}{1+mr} \\
\mu(i+1)G_{n,i,\beta}^* &= \left[\mu i + (n-i)(\Theta_{n,i,\beta}+\rho)\right] G_{n,i,\beta}^* \\
&\quad - (n-i+1)(\Theta_{n,i-1,\beta}+\rho) G_{n,i-1,\beta}^*
\end{aligned}
$$

Epidemic threshold

$$
\left.\frac{\mathrm{d}F}{\mathrm{d}\rho}\right|_{\rho\to 0} > 1
$$

where

$$
F(\rho) \equiv r(\rho)\frac{\sum_m m(m-1)S_m(\rho)}{\sum_m mS_m(\rho)}
$$

The model and its mathematical description

Stationary state

$$S_m^* = \frac{g_m}{1+mr}$$
$$\mu(i+1)G^*_{n,i,\beta} = \left[\mu i + (n-i)(\Theta_{n,i,\beta}+\rho)\right]G^*_{n,i,\beta} - (n-i+1)(\Theta_{n,i-1,\beta}+\rho)G^*_{n,i-1,\beta}$$

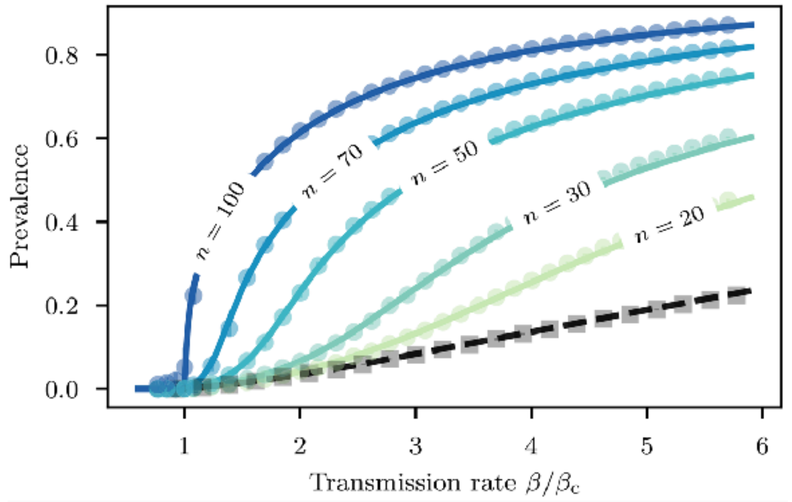
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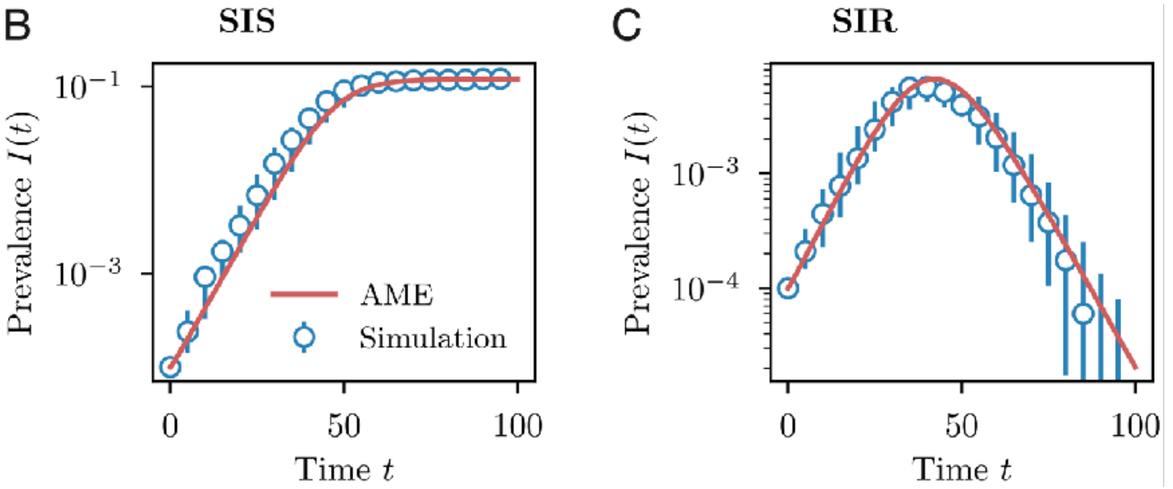
$$\left.\frac{\mathrm{d}F}{\mathrm{d}\rho}\right|_{\rho\to 0} > 1$$

where

$$F(\rho) \equiv r(\rho)\frac{\sum_m m(m-1)S_m(\rho)}{\sum_m mS_m(\rho)}$$

Good agreement between theory and numerical simulations.





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2. Some applications

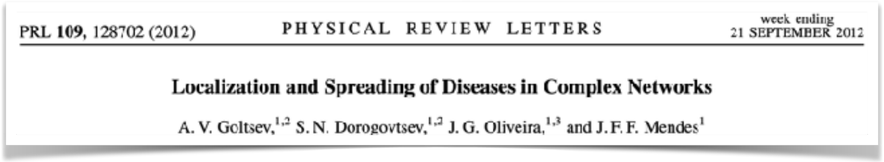
   (a) Mesocopic localization

   (b) Heterogeneous transmission settings

   (c) Context-sensitive behavior

Mesocopic localization

Localization: an epidemic sustains itself via a limited (and localized) number of nodes, traditionnally around hubs or within the inner-most core.


PRL 109, 128702 (2012) PHYSICAL REVIEW LETTERS week ending 21 SEPTEMBER 2012

Localization and Spreading of Diseases in Complex Networks

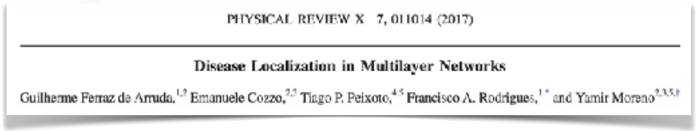
A. V. Goltsev,[1,2] S. N. Dorogovtsev,[1,2] J. G. Oliveira,[1,3] and J. F. F. Mendes[1]


nature > scientific reports > articles > article

Article | Open access | Published: 20 April 2012

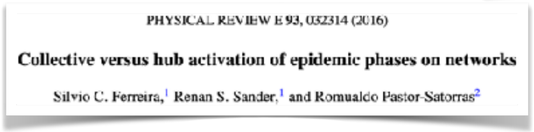
Competing activation mechanisms in epidemics on networks

Claudio Castellano & Romualdo Pastor-Satorras


PHYSICAL REVIEW X 7, 011014 (2017)

Disease Localization in Multilayer Networks

Guilherme Ferraz de Arruda,[1,2] Emanuele Cozzo,[2,3] Tiago P. Peixoto,[4,5] Francisco A. Rodrigues,[1,*] and Yamir Moreno[2,3,5,†]


PHYSICAL REVIEW E 93, 032314 (2016)

Collective versus hub activation of epidemic phases on networks

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IEEE TRANSACTIONS ON NETWORK SCIENCE AND ENGINEERING, VOL. 6, NO. 4, OCTOBER-DECEMBER 2019 983

Network Localization Is Unalterable by Infections in Bursts

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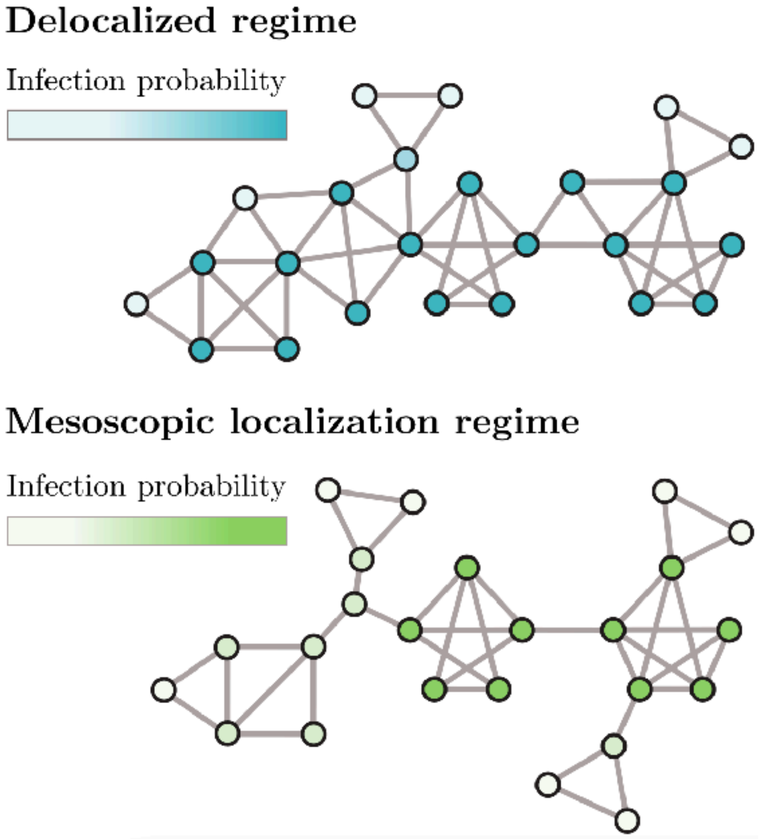
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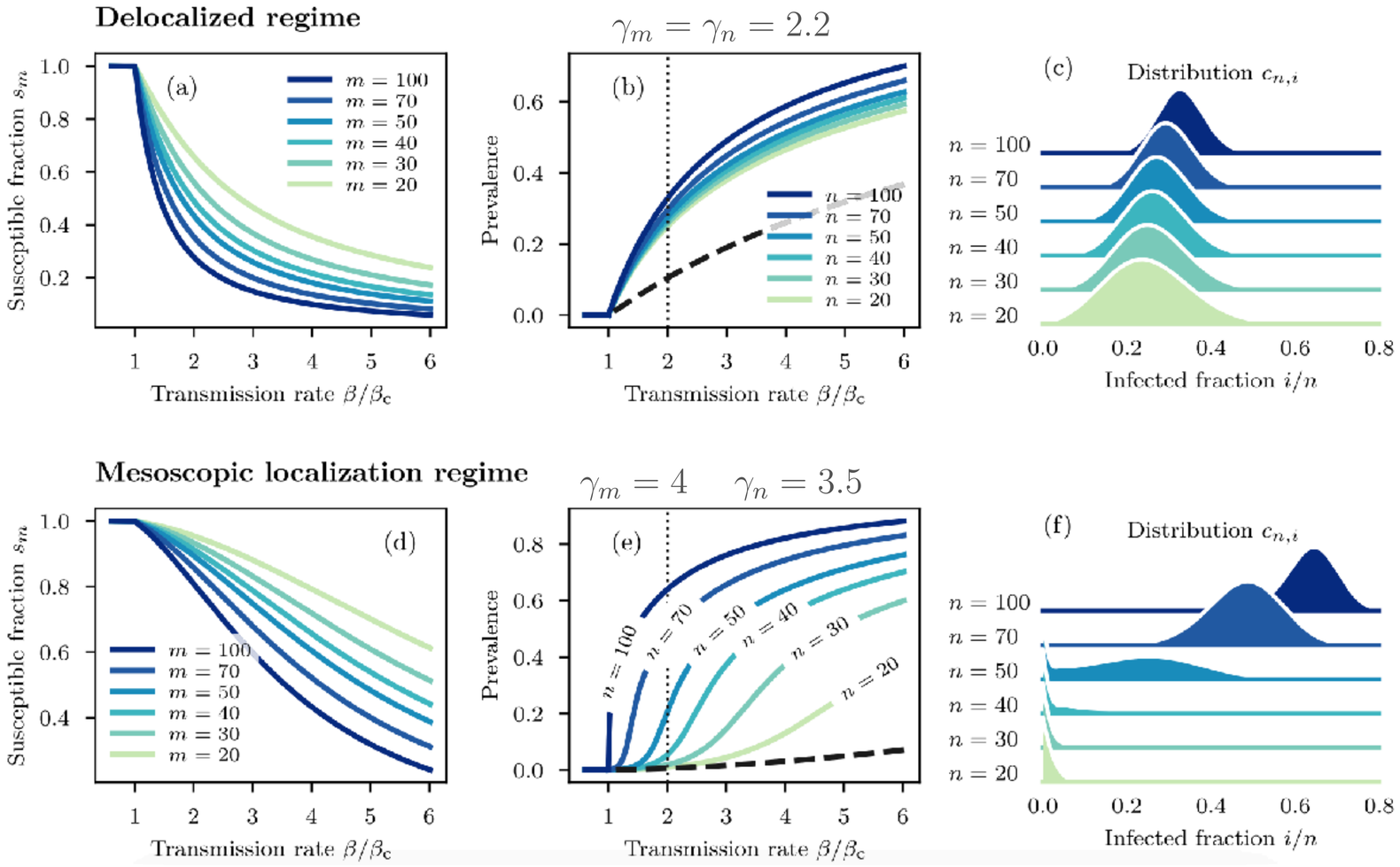
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Mesocopic localization

Message #1: Mesoscopic localization can be observed.

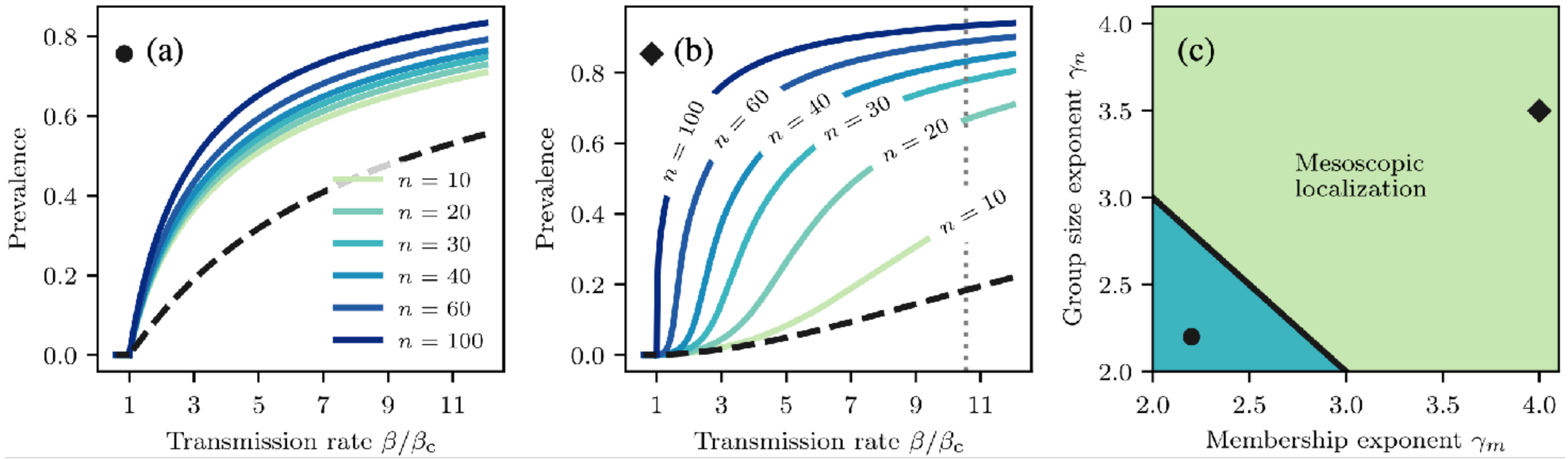
$$\Theta_{n,i,\beta} = \beta\, i\, n^{-\nu}$$
$$\nu \in [0, 1]$$
$$p_n \propto n^{-\gamma_n}$$
$$g_m \propto m^{-\gamma_m}$$
$$\mu = 1$$



Notation: $s_m = S_m$ and $c_{n,i} = G_{n,i}$. β_c is the epidemic threshold.

Mesocopic localization

Message #2: Mesoscopic localization is the rule rather than the exception.

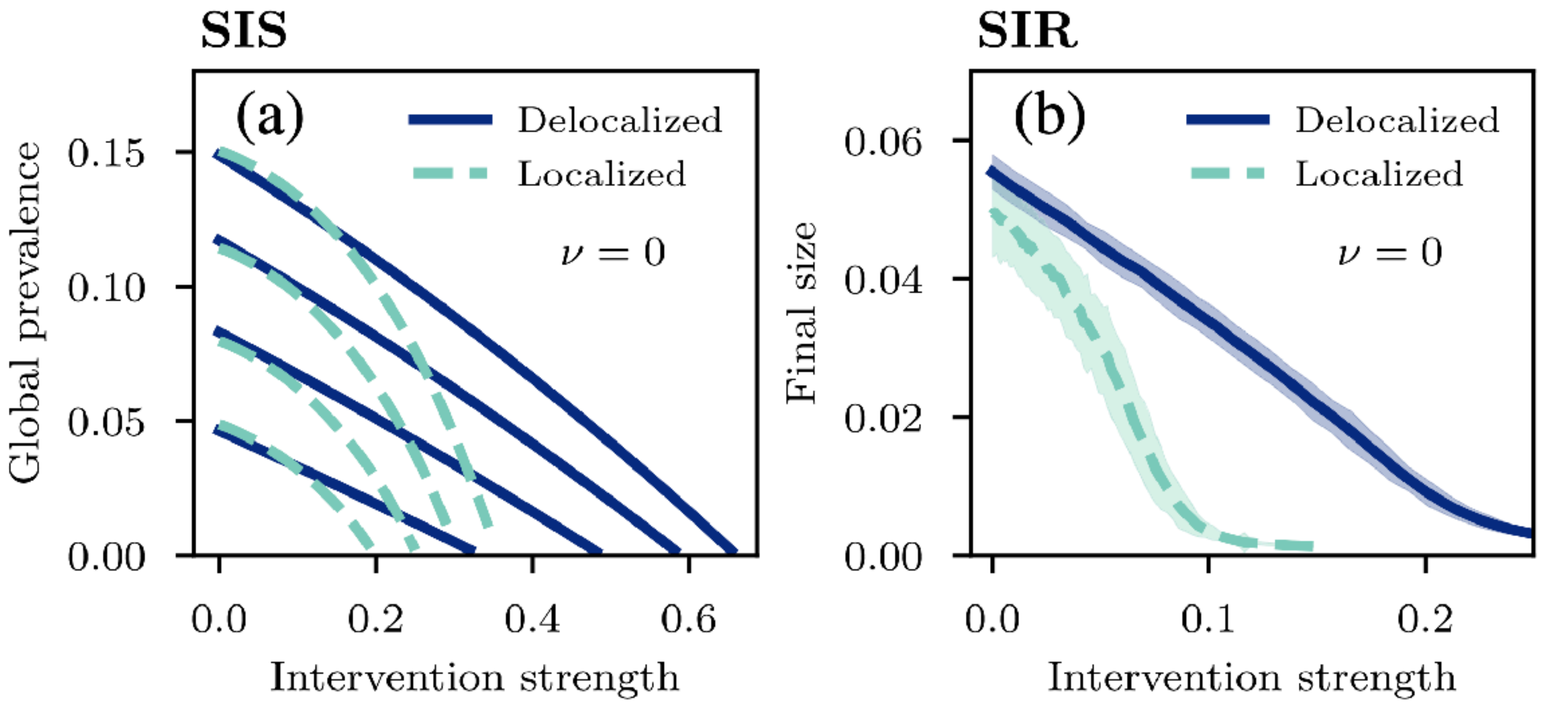


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$$p_n \propto n^{-\gamma_n}$$
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Mesocopic localization

Message #3: Mesoscopic localization offers an opportunity for efficient interventions.

Intervention: enforcing a hard cutoff, $n_{\max}$, on the size of groups;
the lower $n_{\max}$ is, the stronger is the intervention.



$$\Theta_{n,i,\beta} = \beta\, i\, n^{-\nu}$$
$$\nu \in [0, 1]$$
$$p_n \propto n^{-\gamma_n}$$
$$g_m \propto m^{-\gamma_m}$$
$$\mu = 1$$
$$\gamma_m = \gamma_n = 3.5$$

Mesocopic localization

Main takeaways

- The phenomenon of localization extends to mesoscopic structures like groups.
- Mesoscopic localization is the rule rather than the exception.
- Interventions at the group level are more efficient in presence of localization.

PHYSICAL REVIEW LETTERS **126,** 098301 (2021)

Social Confinement and Mesoscopic Localization of Epidemics on Networks

Guillaume St-Onge[1,2] Vincent Thibeault[1,2] Antoine Allard[1,2] Louis J. Dubé,[1,2] and Laurent Hébert-Dufresne[1,3,*]

PHYSICAL REVIEW E **103**, 032301 (2021)

Master equation analysis of mesoscopic localization in contagion dynamics on higher-order networks

Guillaume St-Onge[1,2] Vincent Thibeault[1,2] Antoine Allard[1,2] Louis J. Dubé,[1,2] and Laurent Hébert-Dufresne[1,3,4,*]



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Heterogeneous transmission settings

The context of contacts matters for transmission

- ▷ COVID-19
- ▷ Influenza A
- ▷ STIs
- ▷ . . .

What are the possible effects of ignoring context?


JAMA Insights

Indoor Air Changes and Potential Implications for SARS-CoV-2 Transmission

Joseph G. Allen, DSc, MPH; Andrew M. Ibrahim, MD, MSc


Reviews in Medical Virology

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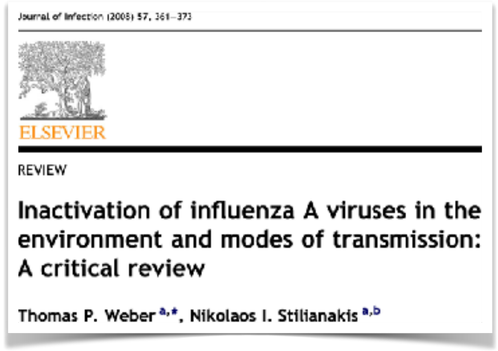
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Original Article

Sex, synchrony, and skin contact: integrating multiple behaviors to assess pathogen transmission risk

Stephan T. Leu, Pratha Sah, Ewa Krzyszczyk, Ann-Marie Jacoby, Janet Mann, and Shweta Bansal


Journal of Infection (2008) 57, 361–373

ELSEVIER

REVIEW

Inactivation of influenza A viruses in the environment and modes of transmission: A critical review

Thomas P. Weber [a,*], Nikolaos I. Stilianakis [a,b]

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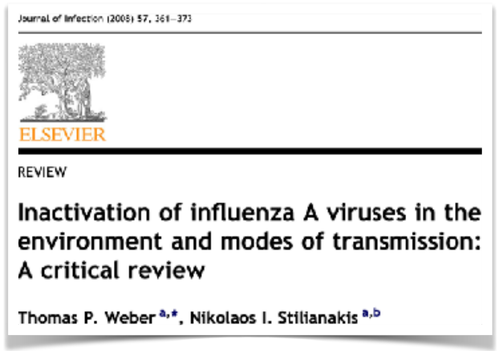
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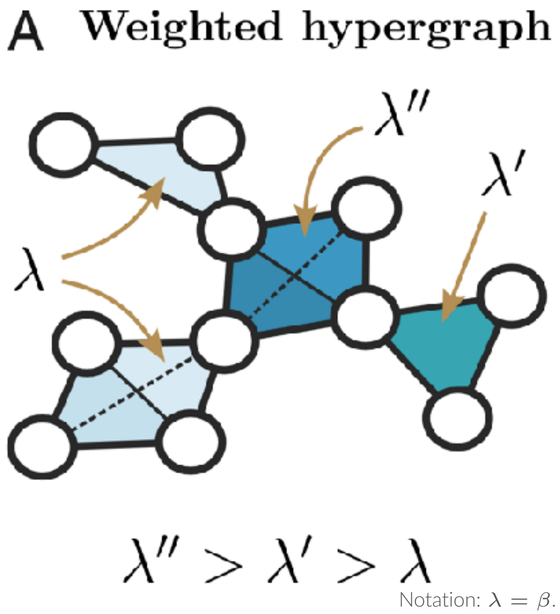
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We include context by allowing each group to have an individual β drawn from a density $f(\beta|n)$.



$$\lambda'' > \lambda' > \lambda$$

Notation: $\lambda = \beta$.

Heterogeneous transmission settings

Detailled description of the dynamics at the groups level with $\Theta_{n,i,\beta} = i\beta$ ("simple contagion")

$$\frac{\mathrm{d}G^{\beta}_{n,i}}{\mathrm{d}t} = \mu(i+1)G^{\beta}_{n,i+1} - \mu i G^{\beta}_{n,i} + (n-i+1)[(i-1)\beta + \rho]G^{\beta}_{n,i-1} - (n-i)[i\beta + \rho]G^{\beta}_{n,i}$$

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Coarse-grained description of the dynamics at the groups level (with $G_{n,i} = \int G^{\beta}_{n,i}\mathrm{d}\beta$)

$$\frac{\mathrm{d}G_{n,i}}{\mathrm{d}t} = \mu(i+1)G_{n,i+1} - \mu i G_{n,i} + (n-i+1)[(i-1)\bar{\beta}_{n,i} + \rho]G_{n,i-1} - (n-i)[i\bar{\beta}_{n,i} + \rho]G_{n,i}$$

where

$$\bar{\beta}_{n,i} = \mathbb{E}[\beta|n,i] = \frac{\int \beta G^{\beta}_{n,i}\mathrm{d}\beta}{\int G^{\beta}_{n,i}\mathrm{d}\beta} \simeq \begin{cases} \dfrac{\mathbb{E}[\beta^i|n]}{\mathbb{E}[\beta^{i-1}|n]} & \text{short term} \\ \mathbb{E}[\beta|n] & \text{long term} \end{cases}$$

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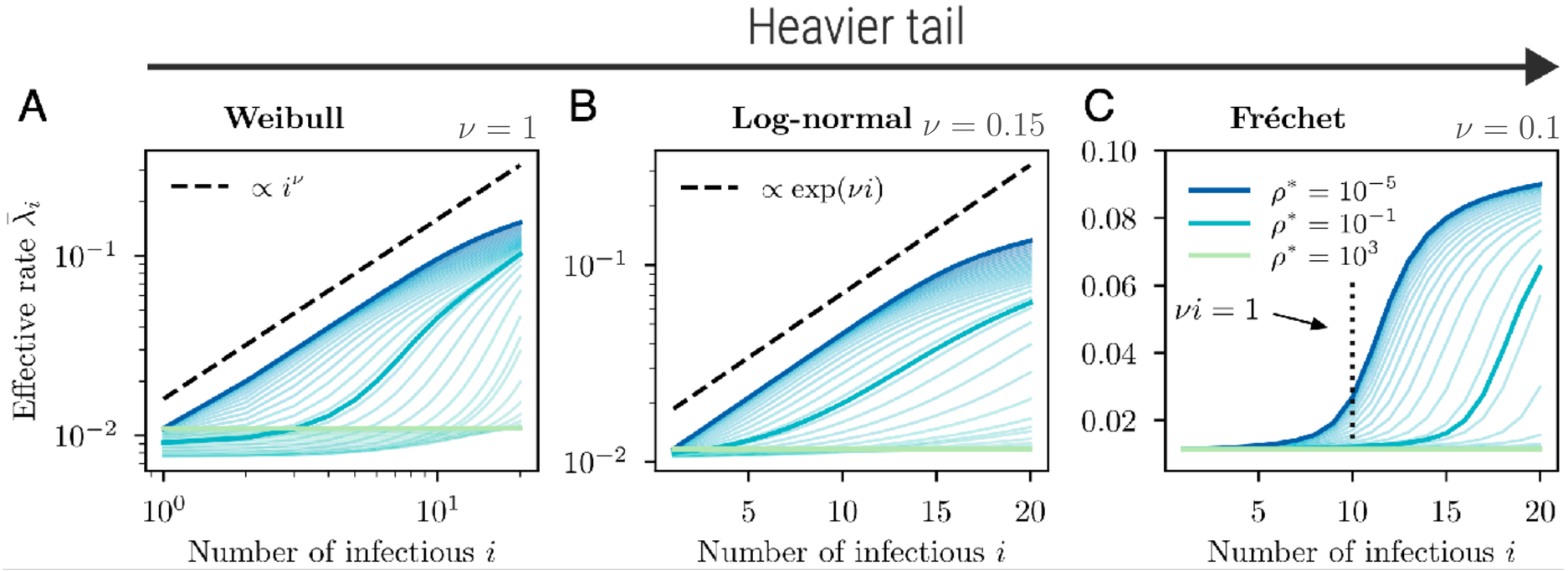
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In its early stage, the epidemic will "see" a transmission rate that depends on i (typically superlinear), akin to "complex contagion".

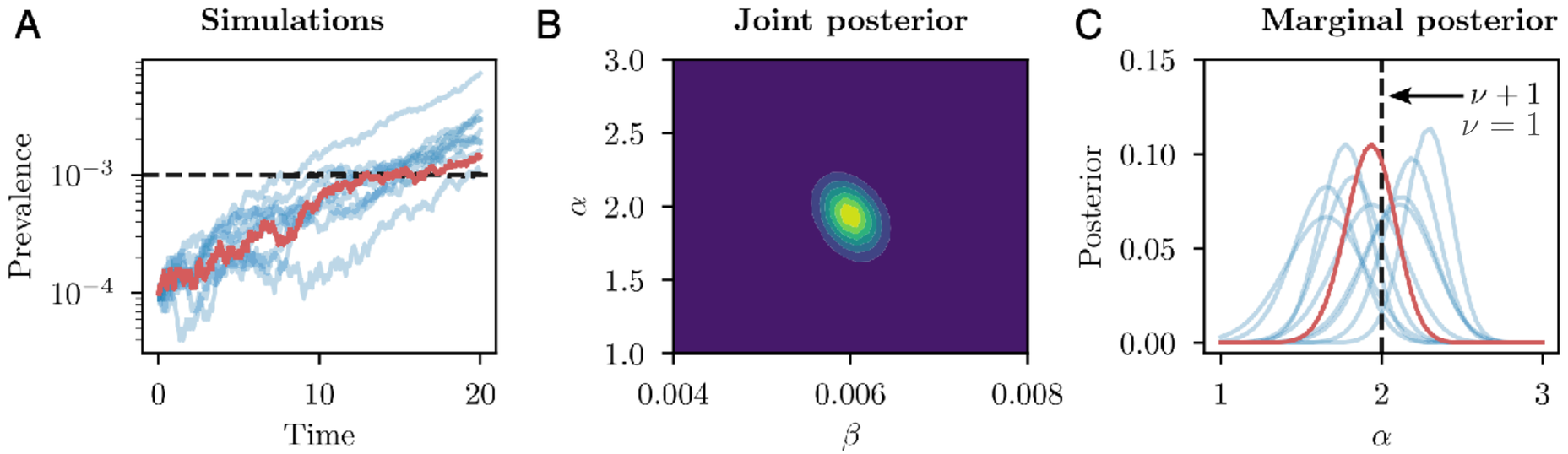
Heterogeneous transmission settings



Notation: $\bar{\lambda}_i = \bar{\beta}_{n,i}$.

Heterogeneous transmission settings

Inference from times series also detects a superlinear transmission rate even if the true underlying contagion is linear.

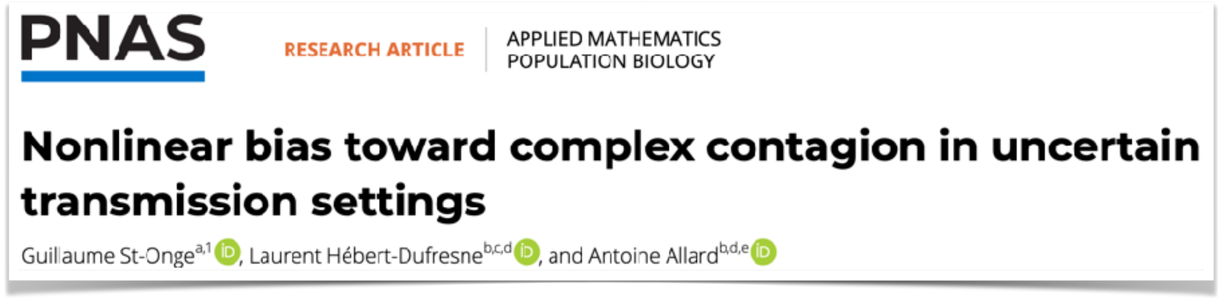


Assumed a transmission rate of the form βi^{α}.

Heterogeneous transmission settings

Main takeaways

- Considering group heterogeneity changes the dynamics.
- Without knowing the local transmission rate, group heterogeneity mimics more complex rules.





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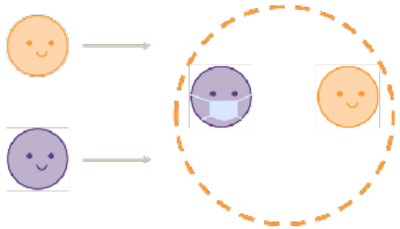
   (c) Context-sensitive behavior

Context-sensitive behavior

Two types of individuals 

- ▷ **Active**: try to reduce transmission risk (e.g. prophylactic measures)
- ▷ **Passive**: do not bother 

Groups are assembled randomly with a prescribed level of homophily

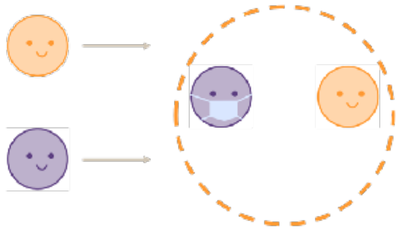


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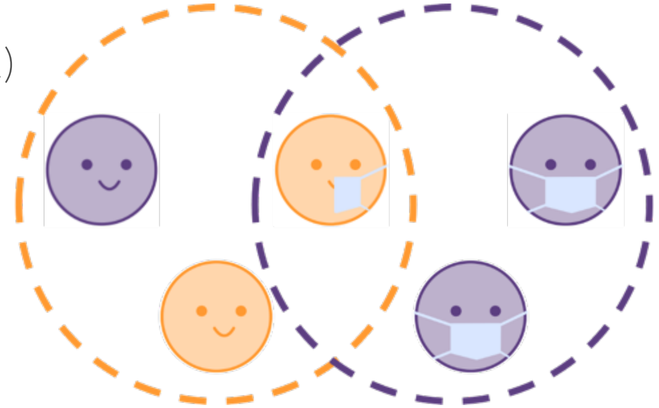
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The transmission rate within each groups depends on its composition

- majority rule (50% + 1)
- arbitrary threshold
- “purity” rule

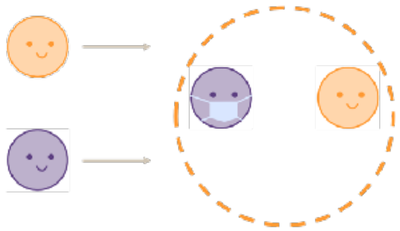


Context-sensitive behavior

Two types of individuals 

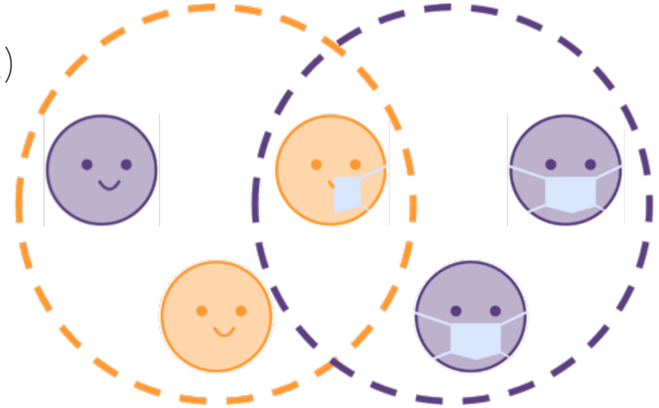
- **Active**: try to reduce transmission risk (e.g. prophylactic measures)
- **Passive**: do not bother 

Groups are assembled randomly with a prescribed level of homophily

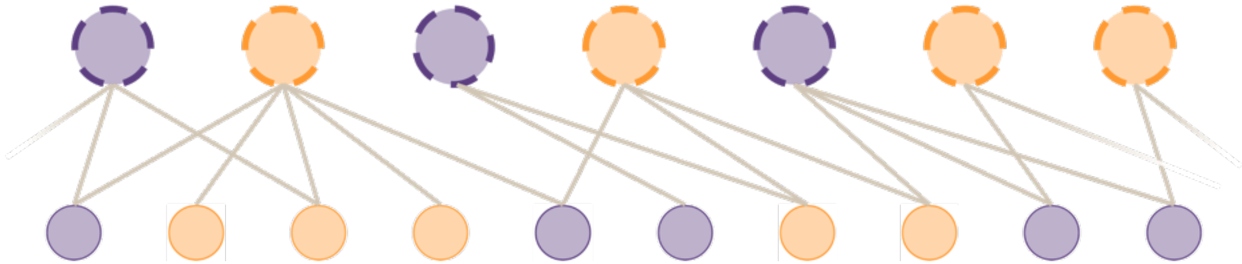


The transmission rate within each groups depends on its composition

- majority rule (50% + 1)
- arbitrary threshold
- “purity” rule

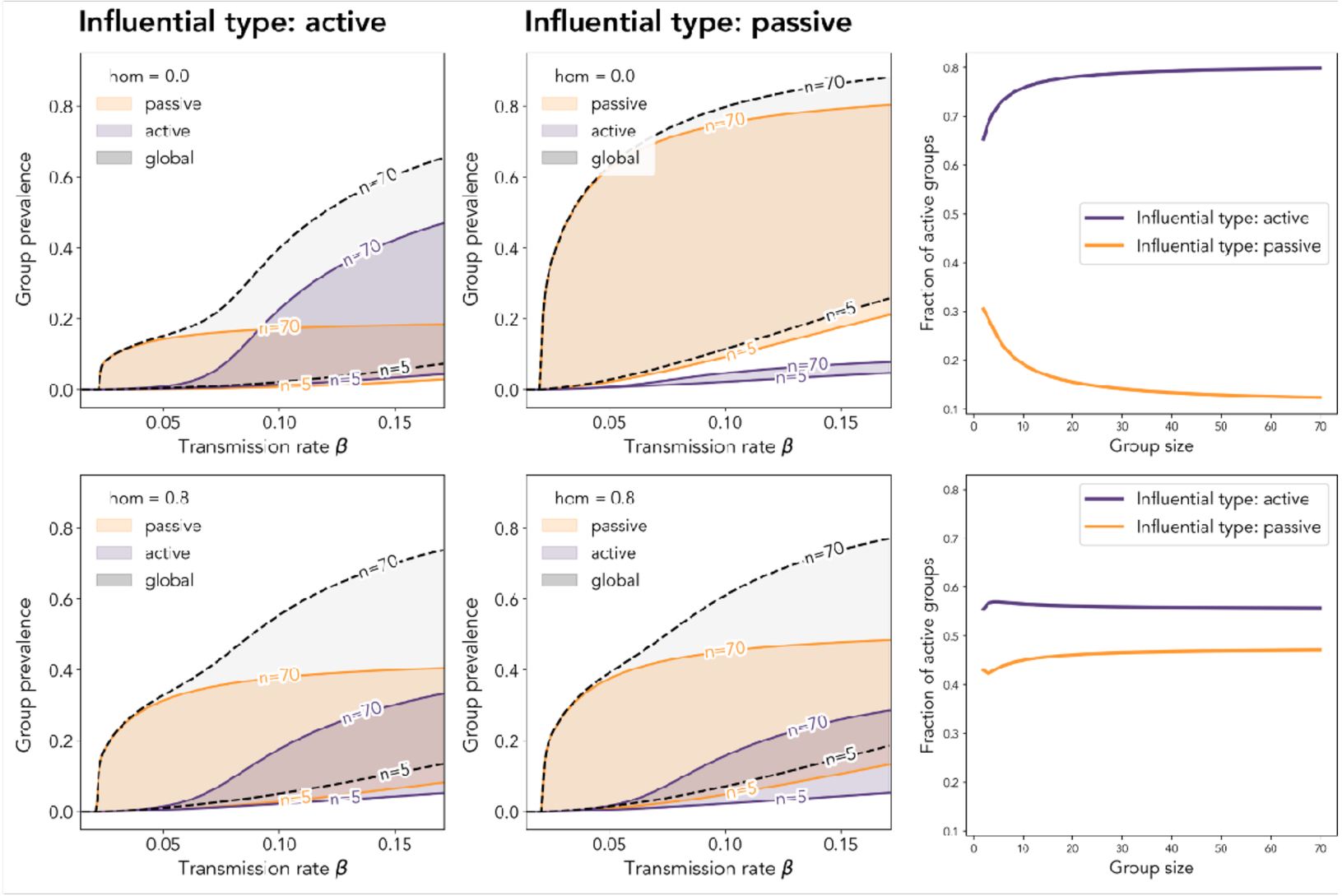


Same framework as before, but with two types of nodes, and with a transmission rate that depends on the composition of groups (composition remains constant over time, for the time being…).



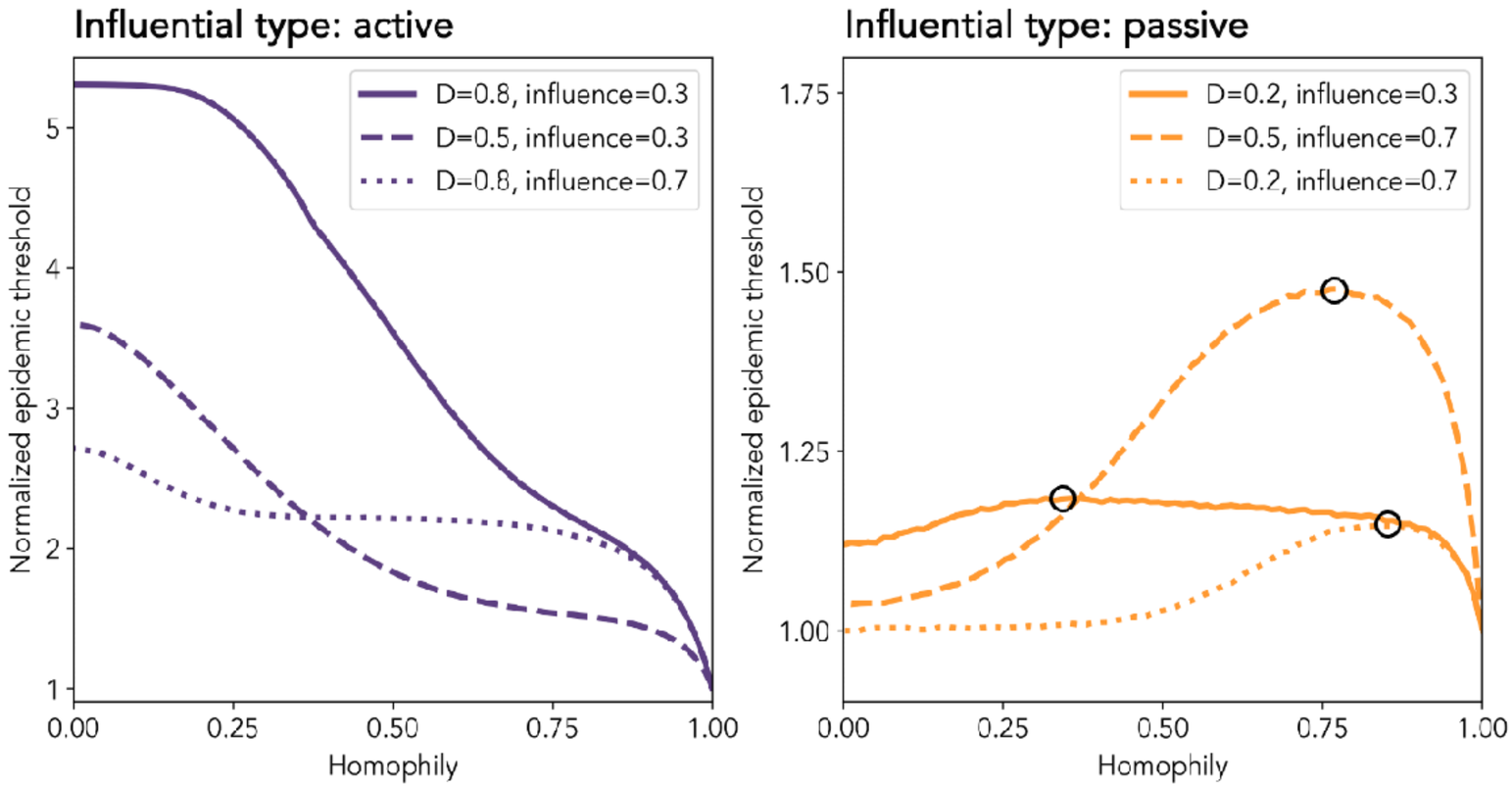
Context-sensitive behavior

Message #1: Increasing homophily favors the least influential group.



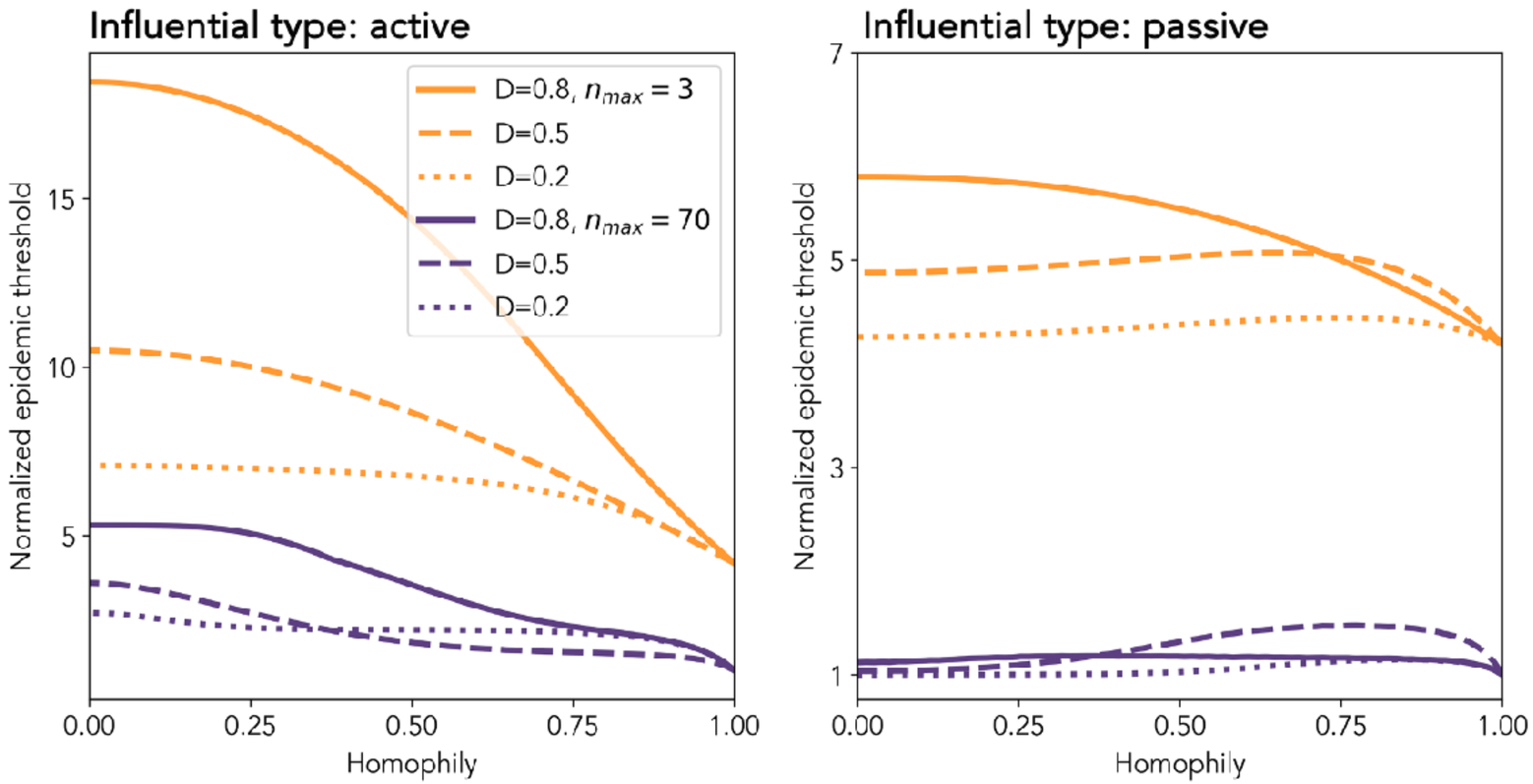
Context-sensitive behavior

Message #2: Epidemic threshold can become a non monotonous function of homophily.



Context-sensitive behavior

Message #3: Accounting for large group sizes may have a qualitative impact on the dynamics.



Context-sensitive behavior

Main takeaways

- Context-sensitive individual behavior is a rich example of higher order interactions.
- Increasing homophily always favors the least influential group.
- Accounting for large group sizes has a significant impact on the dynamics.



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Main takeaways

- Approximate master equations are a flexible framework that allows for the inclusion of complex structure and individual behavior in a tractable epidemiological model
- Shifting the focus from the nodes to the groups provides new insights on interventions, on data analysis, and on factors influencing spreading dynamics.



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Slides

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