Geometry explains the mesoscale organization of empirical directed networks

CONFERENCE ON COMPLEX SYSTEMS 2025

Antoine Allard

Université Laval, Québec, Canada

antoineallard.github.io

@allard.bsky.social

Maximally random graph ensembles

The probability, $P(\mathbb{A})$, for a $N \times N$ adjacency matrix $\mathbb{A} = \{a_{ij}\} \in [0,1]^{\binom{N}{2}}$ that maximizes the entropy subjected to the L constraints ($l = 1, 2, \ldots, L$)

$$S(\{\mathbb{A}\}) = -\sum_{\mathbb{A}} P(\mathbb{A}) \ln P(\mathbb{A}) \qquad \bar{F}_l = \sum_{\mathbb{A}} F_l(\mathbb{A}) P(\mathbb{A})$$

is (α_l being the l-th Lagrange multiplier)

$$P(\mathbb{A}) \propto \exp\left(-\sum_{l=1}^{L} \alpha_l F_l(\mathbb{A})\right) .$$

Phys. Rev. E 70, 066117 (2004)
EPL 81, 28005 (2007)
Phys. Rev. E 68, 026112 (2003)
Phys. Rev. E 86, 026120 (2012)

Maximally random graph ensembles

The probability, $P(\mathbb{A})$, for a $N \times N$ adjacency matrix $\mathbb{A} = \{a_{ij}\} \in [0,1]^{\binom{N}{2}}$ that maximizes the entropy subjected to the L constraints $(l = 1, 2, \ldots, L)$

$$S(\{\mathbb{A}\}) = -\sum_{\mathbb{A}} P(\mathbb{A}) \ln P(\mathbb{A}) \qquad \bar{F}_l = \sum_{\mathbb{A}} F_l(\mathbb{A}) P(\mathbb{A})$$

is (α_l being the l-th Lagrange multiplier)

$$P(\mathbb{A}) \propto \exp\left(-\sum_{l=1}^{L} \alpha_l F_l(\mathbb{A})\right) .$$

Example 1: fixing the expected number of edges

$$\bar{F}_1 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} a_{ij} = M$$

yields the Bernouilli random graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p^{a_{ij}} (1-p)^{1-a_{ij}} \quad \text{with} \quad p = \frac{1}{e^{\alpha_1} + 1} .$$

Phys. Rev. E 70, 066117 (2004)
EPL 81, 28005 (2007)
Phys. Rev. E 68, 026112 (2003)
Phys. Rev. E 86, 026120 (2012)

Maximally random graph ensembles

The probability, $P(\mathbb{A})$, for a $N \times N$ adjacency matrix $\mathbb{A} = \{a_{ij}\} \in [0,1]^{\binom{N}{2}}$ that maximizes the entropy subjected to the L constraints ($l = 1, 2, \ldots, L$)

$$S(\{\mathbb{A}\}) = -\sum_{\mathbb{A}} P(\mathbb{A}) \ln P(\mathbb{A}) \qquad \bar{F}_l = \sum_{\mathbb{A}} F_l(\mathbb{A}) P(\mathbb{A})$$

is (α_l being the l-th Lagrange multiplier)

$$P(\mathbb{A}) \propto \exp\left(-\sum_{l=1}^{L} \alpha_l F_l(\mathbb{A})\right) .$$

Example 1: fixing the expected number of edges

$$\bar{F}_1 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} a_{ij} = M$$

yields the Bernouilli random graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p^{a_{ij}} (1-p)^{1-a_{ij}} \quad \text{with} \quad p = \frac{1}{e^{\alpha_1} + 1} .$$

Example 2: fixing the expected degree sequence

$$\bar{F}_l = \sum_{j=1}^{N} a_{lj} = k_l$$

for $l = 1, \ldots, N$ yields the soft configuration model

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p_{ij}^{a_{ij}} (1-p_{ij})^{1-a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{-(\alpha_i+\alpha_j)} + 1} .$$

Redefining $\kappa_l = \sqrt{\langle\kappa\rangle N} e^{\alpha_l}$ for $l = 1, \ldots, N$ yields the Chung-Lu model

$$p_{ij} = \frac{1}{1 + \frac{\langle\kappa\rangle N}{\kappa_i \kappa_j}} \simeq \frac{\kappa_i \kappa_j}{\langle\kappa\rangle N} .$$

Studying the conditions for which p_{ij} can be factorized informs us on the degree-degree correlations observed in real networks.

Phys. Rev. E 70, 066117 (2004)
EPL 81, 28005 (2007)
Phys. Rev. E 68, 026112 (2003)
Phys. Rev. E 86, 026120 (2012)

Modeling clustering

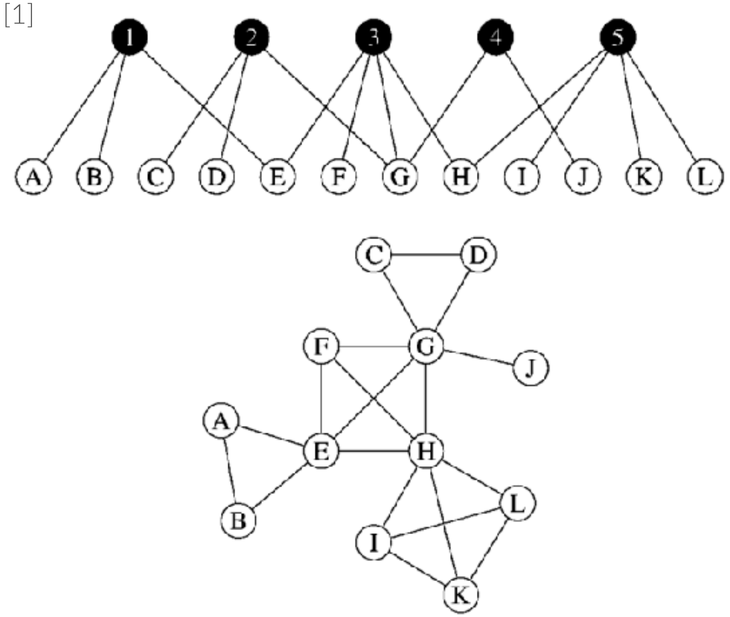
Tricky because clustering consists in three-node interactions while our mathematical tools rely on pairwise interactions either explicitly or implicitly.

Straightforward inclusion of triangles to the maximally random graph ensemble formalism yields unwanted behavior (ex.: triangle agglutination in the Strauss model [6]).

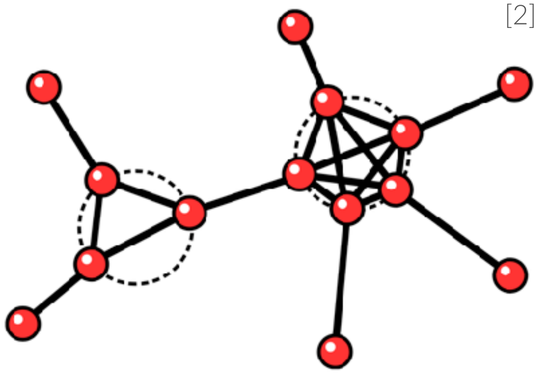
Most models therefore assume

- ▷ an underlying tree-like structure
- ▷ that the networks are dense

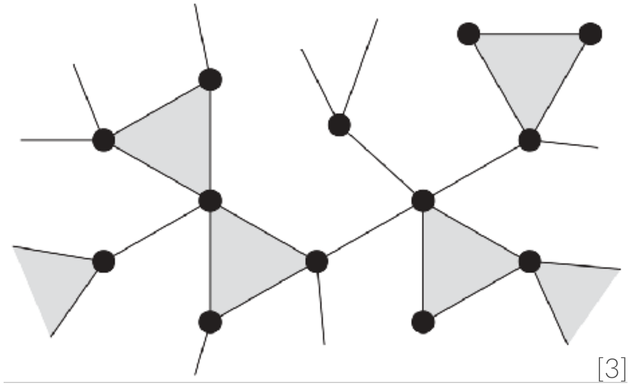
[1]



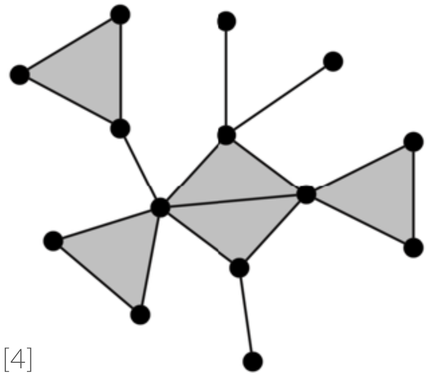
[2]



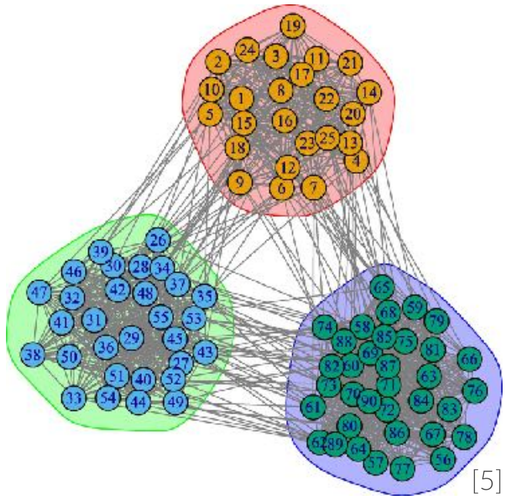
[3]



[4]



[5]



[1] Phys. Rev. E 68, 026121 (2003)
[2] Phys. Rev. E 80, 036107 (2009)
[3] Phys. Rev. Lett. 103, 058701 (2009)
[4] Phys. Rev. E 82, 066118 (2010)
[5] Appl. Netw. Sci. 4, 122 (2019)
[6] Phys. Rev. E 72, 026136 (2005)

Modeling clustering

Idea: Assume that the vertices are embedded in a metric space and that any two vertices are connected with a probability that is a decreasing function of the distance between them.

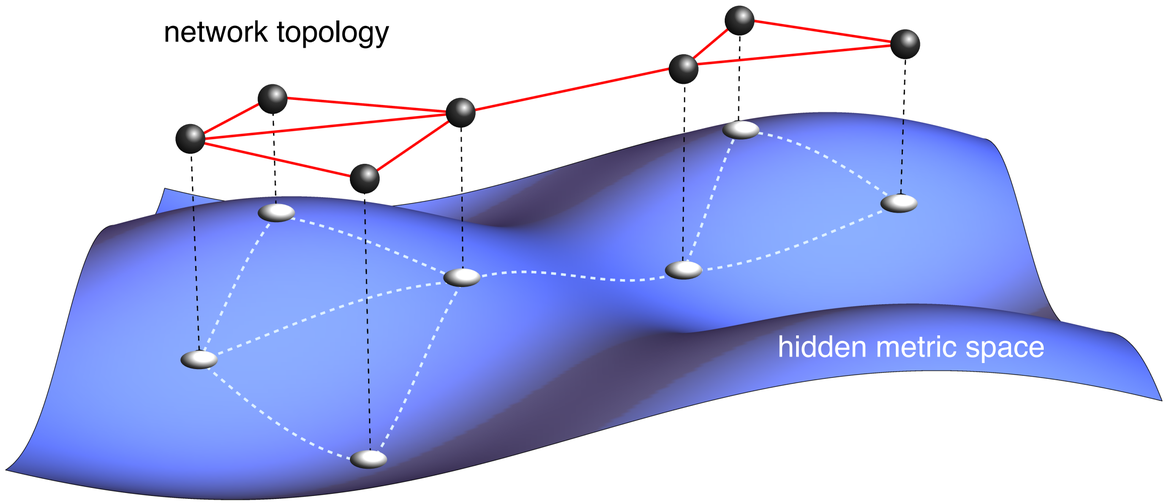
Courtesy of M. Boguñá

Modeling clustering

Idea: Assume that the vertices are embedded in a metric space and that any two vertices are connected with a probability that is a decreasing function of the distance between them.

Properties of any metric space:

Identity of indiscernibles	$d(x,y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x,y) \geq 0$
Symmetry	$d(x,y) = d(y,x)$
Triangle inequality	$d(x,y) \leq d(x,z) + d(z,y)$



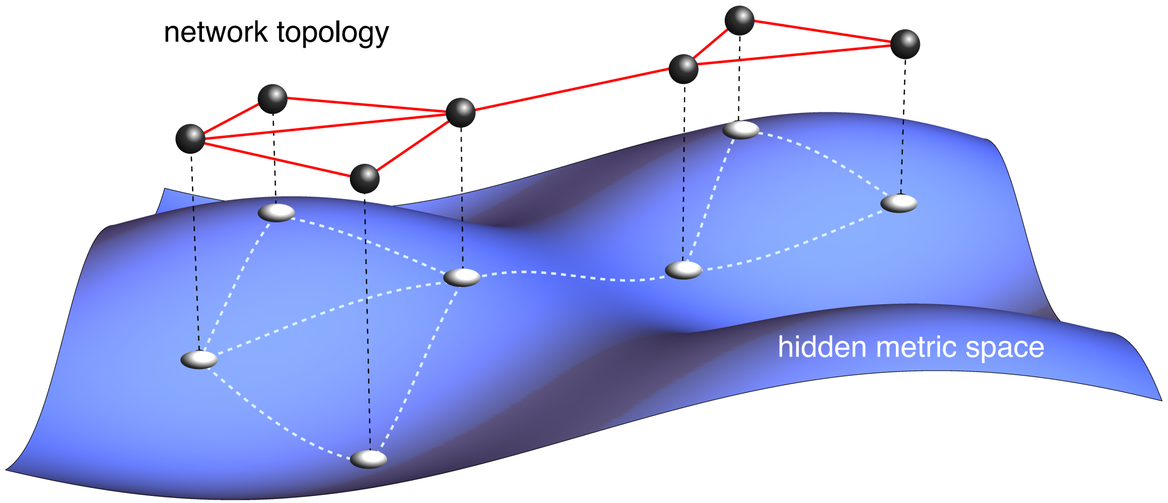
Courtesy of M. Boguñá

Modeling clustering

Idea: Assume that the vertices are embedded in a metric space and that any two vertices are connected with a probability that is a decreasing function of the distance between them.

Properties of any metric space:

Identity of indiscernibles	$d(x,y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x,y) \geq 0$
Symmetry	$d(x,y) = d(y,x)$
Triangle inequality	$d(x,y) \leq d(x,z) + d(z,y)$



Courtesy of M. Boguñá

Henceforth we consider that there is a distance matrix $\mathbb{X} = \{x_{ij}\}$ where x_{ij} is the distance between vertices i and j in the D-dimensional space.

Maximally random geometric graph ensembles

Example 3: fixing the expected number of edges and the expected total energy

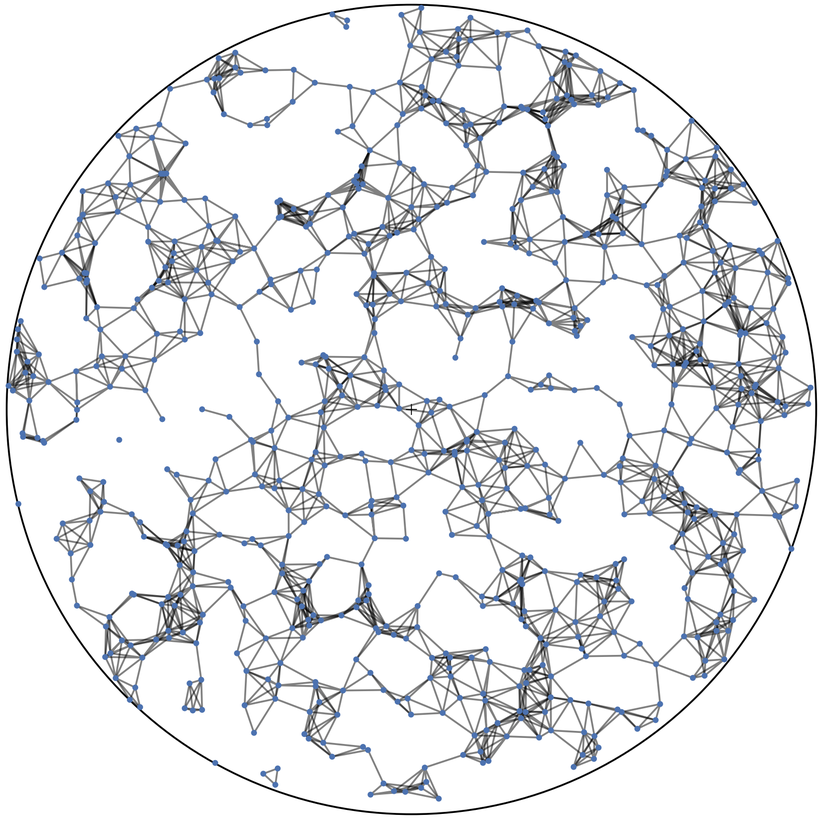
$$\bar{F}_1 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} a_{ij} = M$$

$$\bar{F}_2 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \varepsilon_{ij} a_{ij} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} f(x_{ij}) a_{ij} = E$$

yields the homogeneous random geometric graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p_{ij}^{a_{ij}} (1 - p_{ij})^{1-a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{\beta(\varepsilon_{ij} - \mu)} + 1} \, .$$

The graphs will be sparse, highly clustered and small-world iif $f(x_{ij}) \sim \ln x_{ij}$ and $\beta \in [D, D+2]$.



Maximally random geometric graph ensembles

Example 4: fixing the expected degree sequence and the expected total energy

$$\bar{F}_l = \sum_{j=1}^{N} a_{lj} = k_l \quad (l = 1, \ldots, N)$$

$$\bar{F}_{N+1} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \varepsilon_{ij} a_{ij} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} f(x_{ij}) a_{ij} = E$$

yields the heterogeneous random geometric graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p_{ij}^{a_{ij}} (1 - p_{ij})^{1 - a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{\beta \varepsilon_{ij} - \alpha_i - \alpha_j} + 1} \, .$$

The graphs will be sparse, highly clustered, small-world and devoid of non-structural degree-degree correlation iif $f(x_{ij}) = \ln x_{ij}$ and $\beta \in [D, D+2]$[a]. Redefining $\alpha_l = -(\beta/D) \ln(\sqrt{\mu} \kappa_l)$ yields

$$p_{ij} = \frac{1}{e^{\beta(\varepsilon_{ij} - \mu)} + 1} \quad \text{with} \quad \varepsilon_{ij} = \ln \left(\frac{x_{ij}}{(\kappa_i \kappa_j)^{\frac{1}{D}}} \right) \, .$$

[a] No upper bound if expected degree sequence is scale-free.

Phys. Rev. E 80, 035101 (2009)
Phys. Rev. E 82, 036106 (2010)
Phys. Rev. Research 2, 023040 (2020)

Maximally random geometric graph ensembles

Example 4: fixing the expected degree sequence and the expected total energy

$$\bar{F}_l = \sum_{j=1}^{N} a_{lj} = k_l \quad (l = 1, \ldots, N)$$

$$\bar{F}_{N+1} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \varepsilon_{ij} a_{ij} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} f(x_{ij}) a_{ij} = E$$

yields the heterogeneous random geometric graph ensemble

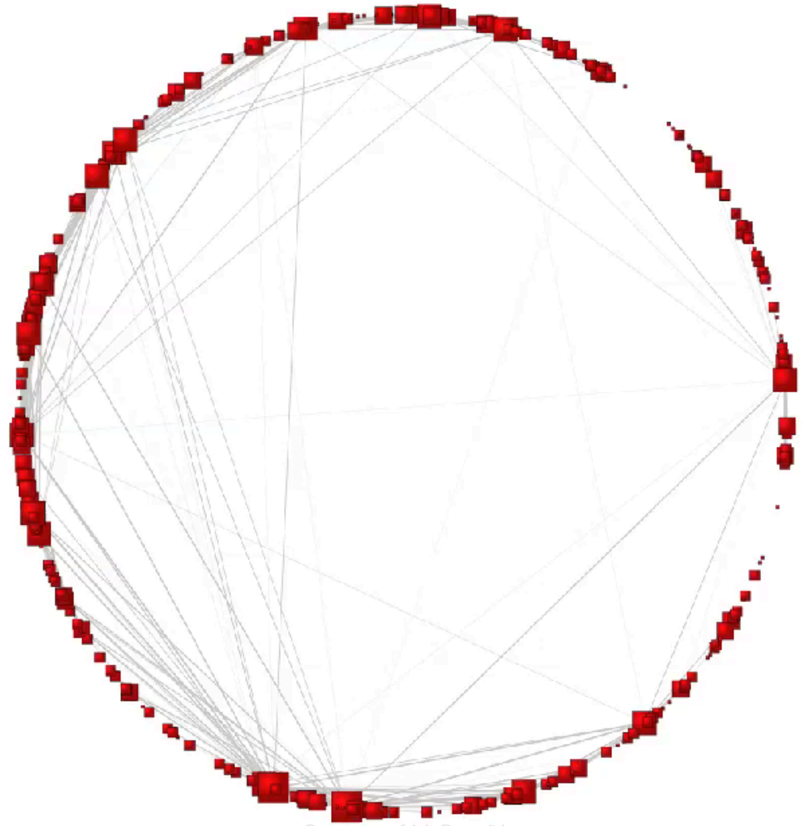
$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p_{ij}^{a_{ij}} (1 - p_{ij})^{1 - a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{\beta \varepsilon_{ij} - \alpha_i - \alpha_j} + 1} \, .$$

The graphs will be sparse, highly clustered, small-world and devoid of non-structural degree-degree correlation iif $f(x_{ij}) = \ln x_{ij}$ and $\beta \in [D, D+2]$[a]. Redefining $\alpha_l = -(\beta/D) \ln(\sqrt{\mu} \kappa_l)$ yields

$$p_{ij} = \frac{1}{e^{\beta(\varepsilon_{ij} - \mu)} + 1} \quad \text{with} \quad \varepsilon_{ij} = \ln \left(\frac{x_{ij}}{(\kappa_i \kappa_j)^{\frac{1}{D}}} \right) \, .$$

[a] No upper bound if expected degree sequence is scale-free.

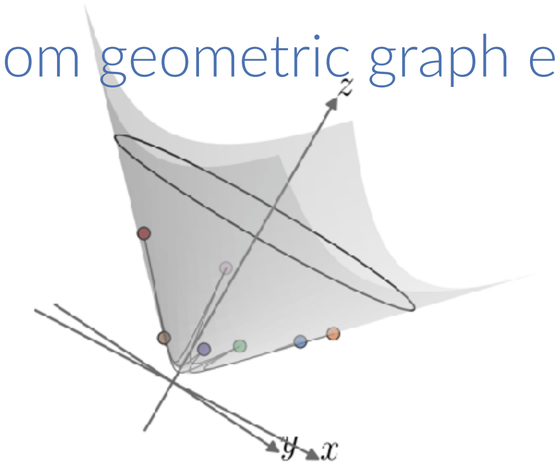
When the geometry is a D-dimensional sphere, $\mathbb{S}^D$ the model can be mapped to a purely geometric model in hyperbolic space $\mathbb{H}^{D+1}$.



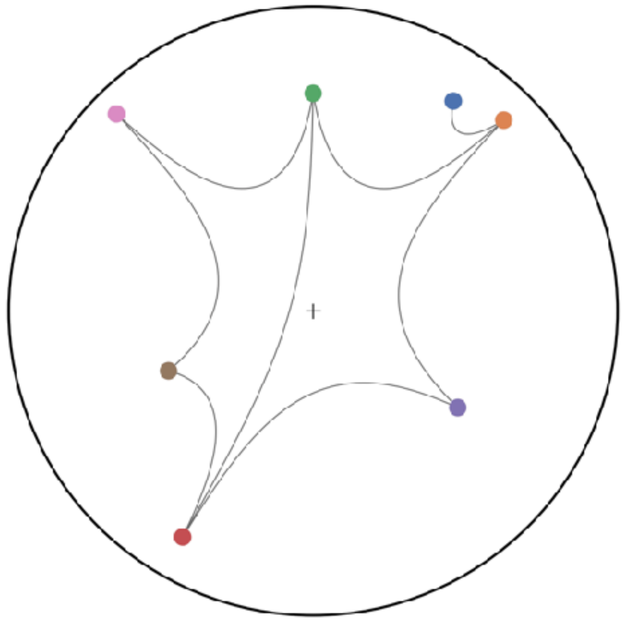
Courtesy of M. Boguñá

Phys. Rev. E 80, 035101 (2009)
Phys. Rev. E 82, 036106 (2010)
Phys. Rev. Research 2, 023040 (2020)

Maximally random geometric graph ensembles

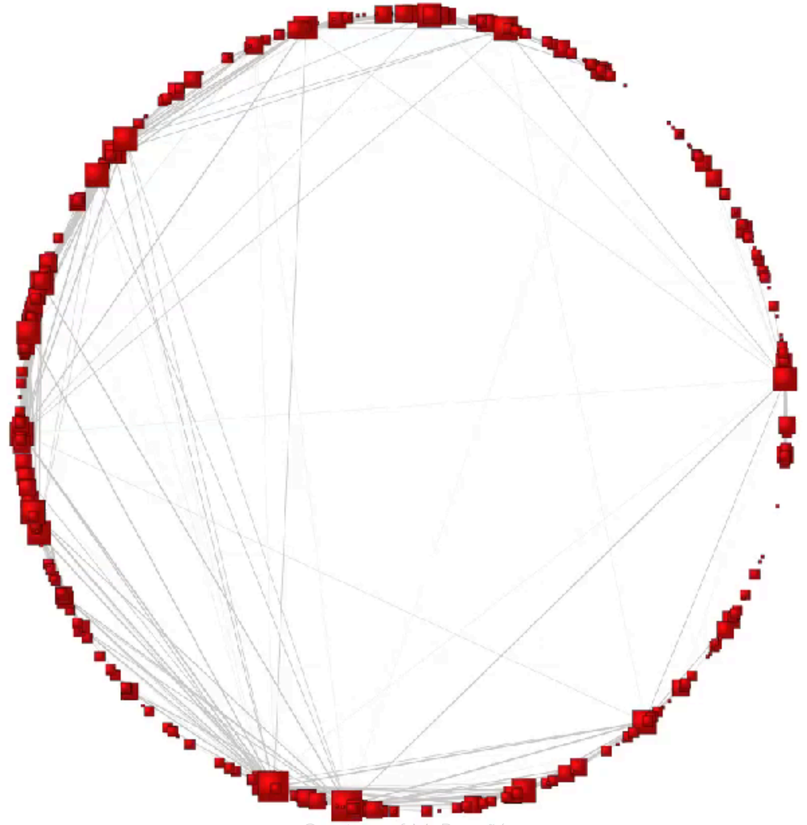


hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)

When the geometry is a D-dimensional sphere, $\mathbb{S}^D$ the model can be mapped to a purely geometric model in hyperbolic space $\mathbb{H}^{D+1}$.

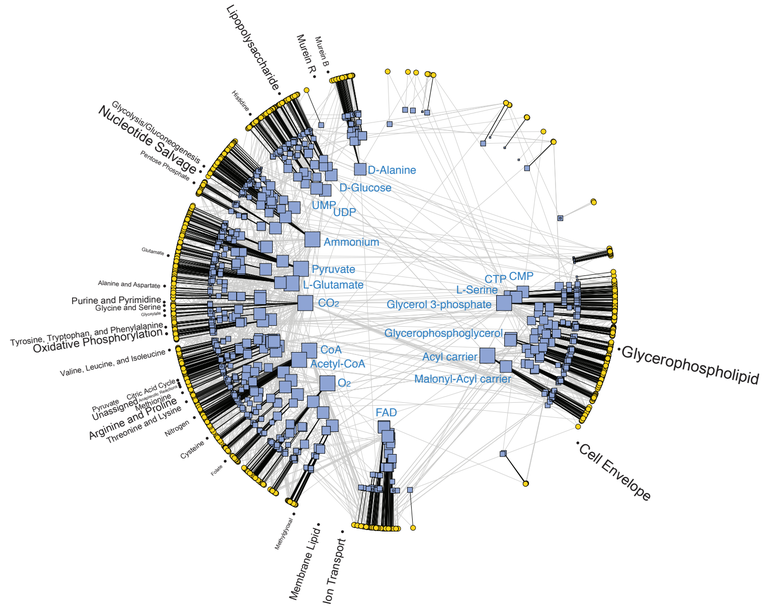


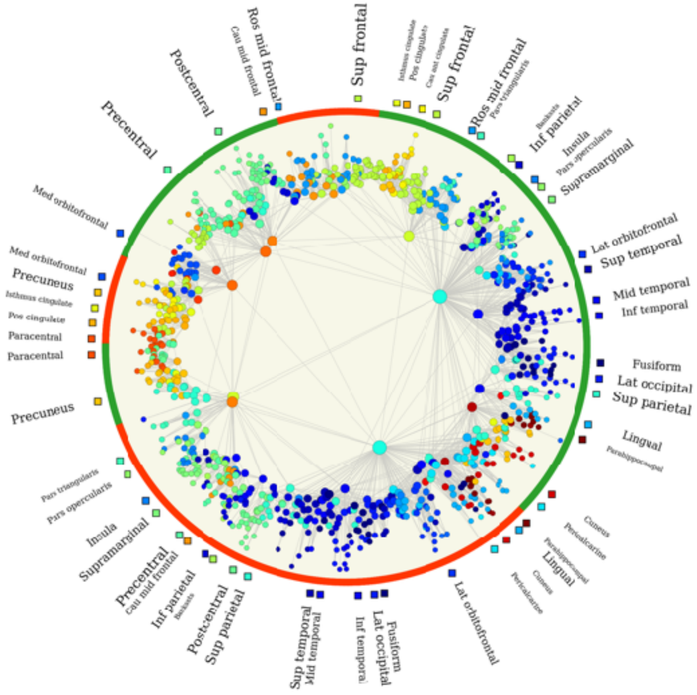
Courtesy of M. Boguñá

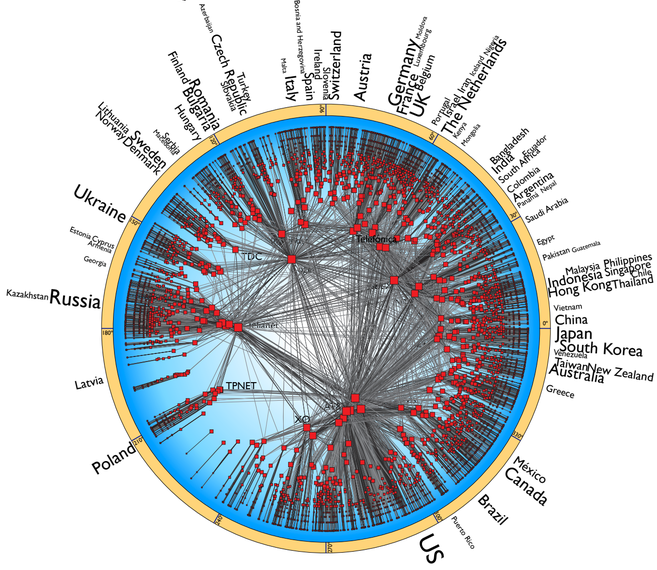
Phys. Rev. E 80, 035101 (2009)
Phys. Rev. E 82, 036106 (2010)
Phys. Rev. Research 2, 023040 (2020)

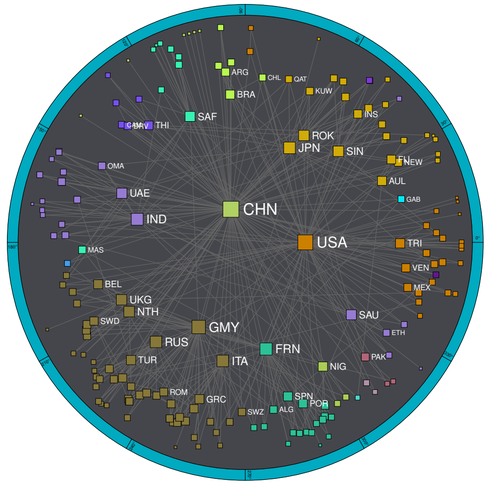
A powerful and versatile framework

- Amenable to many analytical calculations [1,2,3]
- Generalizable to weighted [5], bipartite [6,7,8], multiplex [9,10] and growing [11] networks
- Geometrical interpretation of preferential attachment [11]
- Parsimonious explanation of self-similarity [4]
- Generalizable to networks with community structure [12,13,14]
- Mapping of real complex networks unto hyperbolic space [15,16]
  - Reproduction of additional properties than the ones used to fit the parameters [15].
  - Identification of biochemical pathways in E. Coli [8]
  - Efficient Internet routing protocols [17]
  - Organization of the human connectome [18,20]
  - Self-similar architecture [19]
  - Evolution of hierarchy in international trade [21]
  - …
- …









[1] Phys. Rev. E 80, 035101 (2009)
[2] Phys. Rev. E 82, 036106 (2010)
[3] Phys. Rev. Research 2, 023040 (2020)
[4] Phys. Rev. Lett. 100, 078701 (2008)
[5] Nat. Commun. 8, 14103 (2017)
[6] Phys. Rev. E 84, 026114 (2011)
[7] Phys. Rev. E 95, 032309 (2017)
[8] Mol. Biosyst. 8, 843 (2012)
[9] Nat. Phys. 12, 1076 (2016)
[10] Phys. Rev. Lett. 118, 218301 (2017)
[11] Nature 489, 537 (2012)
[12] Sci. Rep. 5, 9421 (2015)
[13] J. Stat. Phys. 173, 775 (2018)
[14] New J. Phys. 20, 052002 (2018)
[15] New J. Phys. 21, 123033 (2019)
[16] Nat. Commun. 8, 1615 (2017)
[17] Nat. Commun. 1, 62 (2010)
[18] PNAS 117, 20244 (2020)
[19] Nat. Phys. 14, 583 (2018)
[20] PLOS Comput. Biol. 16, e1007584 (2020)
[21] Sci. Rep. 6, 33441 (2016)

A powerful and versatile framework

- Amenable to many analytical calculations [1,2,3]
- Generalizable to weighted [5], bipartite [6,7,8], multiplex [9,10] and growing [11] networks
- Geometrical interpretation of preferential attachment [11]
- Parsimonious explanation of self-similarity [4]
- Generalizable to networks with community structure [12,13,14]
- Mapping of real complex networks unto hyperbolic space [15,16]
  - Reproduction of additional properties than the ones used to fit the parameters [15].
  - Identification of biochemical pathways in E. Coli [8]
  - Efficient Internet routing protocols [17]
  - Organization of the human connectome [18,20]
  - Self-similar architecture [19]
  - Evolution of hierarchy in international trade [21]
  - …
- …

Review Article | Published: 29 January 2021

Network geometry

Marián Boguñá, Ivan Bonamassa, Manlio De Domenico ✉, Shlomo Havlin, Dmitri Krioukov & M. Ángeles Serrano

Nature Reviews Physics **3**, 114–135 (2021)

[1] Phys. Rev. E 80, 035101 (2009)
[2] Phys. Rev. E 82, 036106 (2010)
[3] Phys. Rev. Research 2, 023040 (2020)
[4] Phys. Rev. Lett. 100, 078701 (2008)
[5] Nat. Commun. 8, 14103 (2017)
[6] Phys. Rev. E 84, 026114 (2011)
[7] Phys. Rev. E 95, 032309 (2017)
[8] Mol. Biosyst. 8, 843 (2012)
[9] Nat. Phys. 12, 1076 (2016)
[10] Phys. Rev. Lett. 118, 218301 (2017)
[11] Nature 489, 537 (2012)
[12] Sci. Rep. 5, 9421 (2015)
[13] J. Stat. Phys. 173, 775 (2018)
[14] New J. Phys. 20, 052002 (2018)
[15] New J. Phys. 21, 123033 (2019)
[16] Nat. Commun. 8, 1615 (2017)
[17] Nat. Commun. 1, 62 (2010)
[18] PNAS 117, 20244 (2020)
[19] Nat. Phys. 14, 583 (2018)
[20] PLOS Comput. Biol. 16, e1007584 (2020)
[21] Sci. Rep. 6, 33441 (2016)

Three challenges in modeling directed networks

Properties of any metric space

Identity of indiscernibles	$d(x,y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x,y) \geq 0$
Symmetry	$d(x,y) = d(y,x)$
Triangle inequality	$d(x,y) \leq d(x,z) + d(z,y)$

with community structure[29,32,115]. The model has yet to be extended to directed networks, because it is not clear how to reconcile the intrinsic symmetry of metric distances with asymmetric interactions among nodes.

Nat. Rev. Phys. 3, 114 (2021)

Three challenges in modeling directed networks

Properties of any metric space

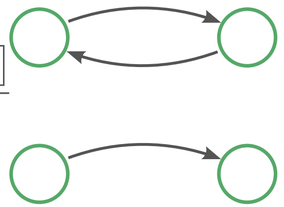
Identity of indiscernibles	$d(x,y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x,y) \geq 0$
Symmetry	$d(x,y) = d(y,x)$
Triangle inequality	$d(x,y) \leq d(x,z) + d(z,y)$

with community structure[29,32,115]. The model has yet to be extended to directed networks, because it is not clear how to reconcile the intrinsic symmetry of metric distances with asymmetric interactions among nodes.

Nat. Rev. Phys. 3, 114 (2021)

Reciprocity : cycles of length 2

$$r = \frac{L^{\leftrightarrow}}{L} = \frac{\text{[number of reciprocal links]}}{\text{[number of links]}}$$



Three challenges in modeling directed networks

Properties of any metric space

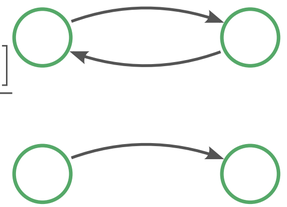
Identity of indiscernibles	$d(x,y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x,y) \geq 0$
Symmetry	$d(x,y) = d(y,x)$
Triangle inequality	$d(x,y) \leq d(x,z) + d(z,y)$

with community structure[29,32,115]. The model has yet to be extended to directed networks, because it is not clear how to reconcile the intrinsic symmetry of metric distances with asymmetric interactions among nodes.

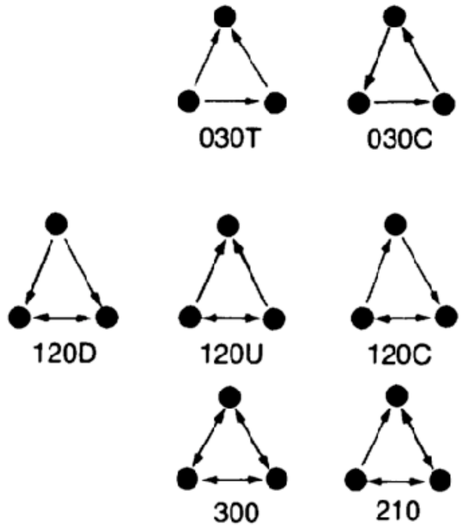
Nat. Rev. Phys. 3, 114 (2021)

Reciprocity : cycles of length 2

$$r = \frac{L^{\leftrightarrow}}{L} = \frac{\text{[number of reciprocal links]}}{\text{[number of links]}}$$

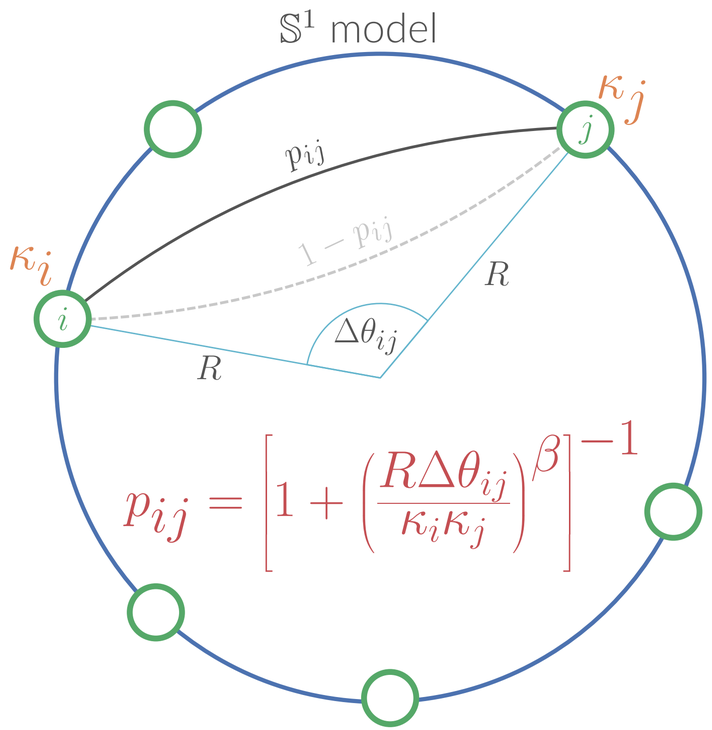


Clustering : 7 cycles of length 3

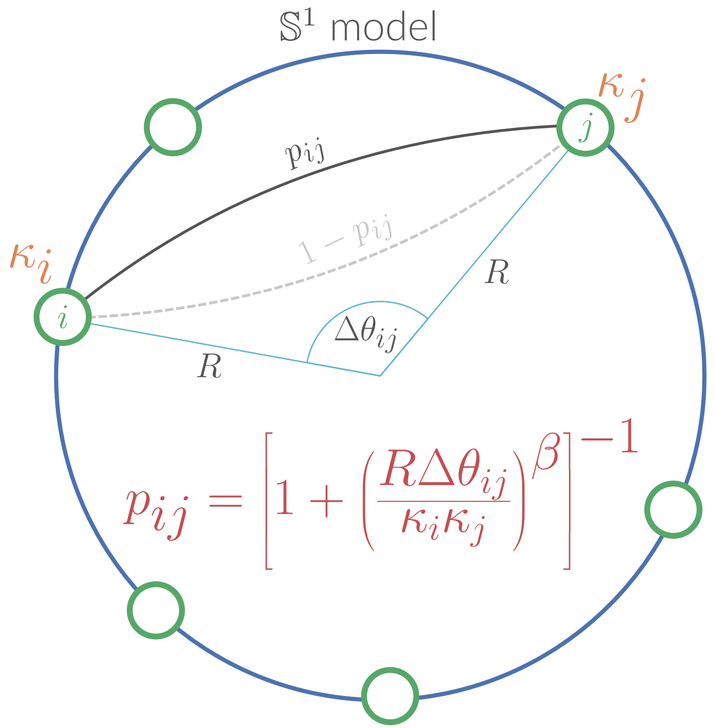


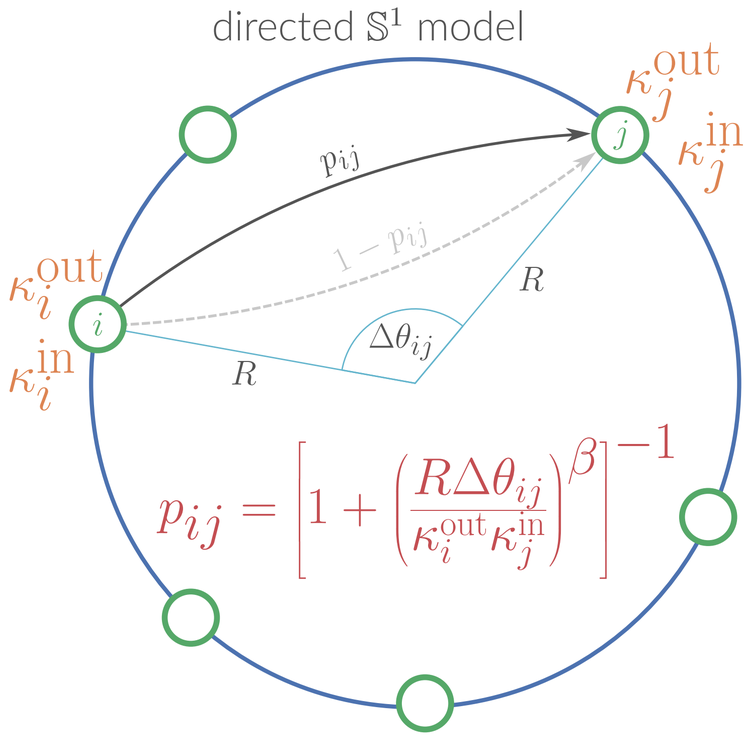
Adapted from Holland & Leinhardt. Local Structure in Social Networks. Sociol. Methodol., 7, 1–45 (1976)

The directed $\mathbb{S}^1$ model: A straightforward generalization



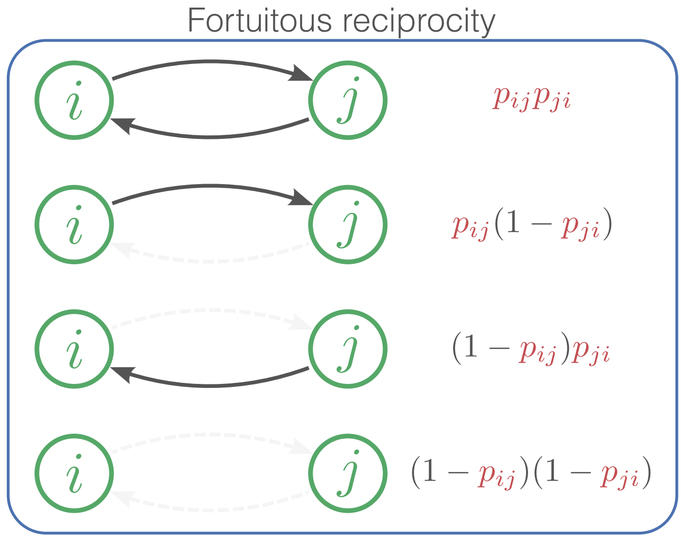
The directed $\mathbb{S}^1$ model: A straightforward generalization





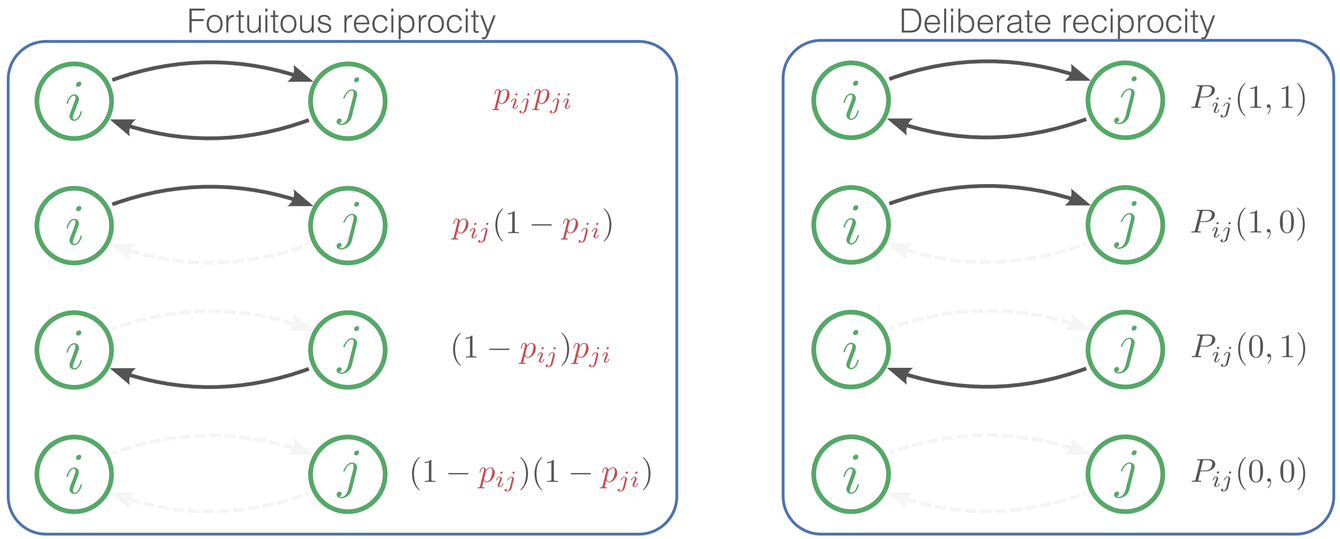
The directed-reciprocal $\mathbb{S}^1$ model: A new link between connections and distances

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j.



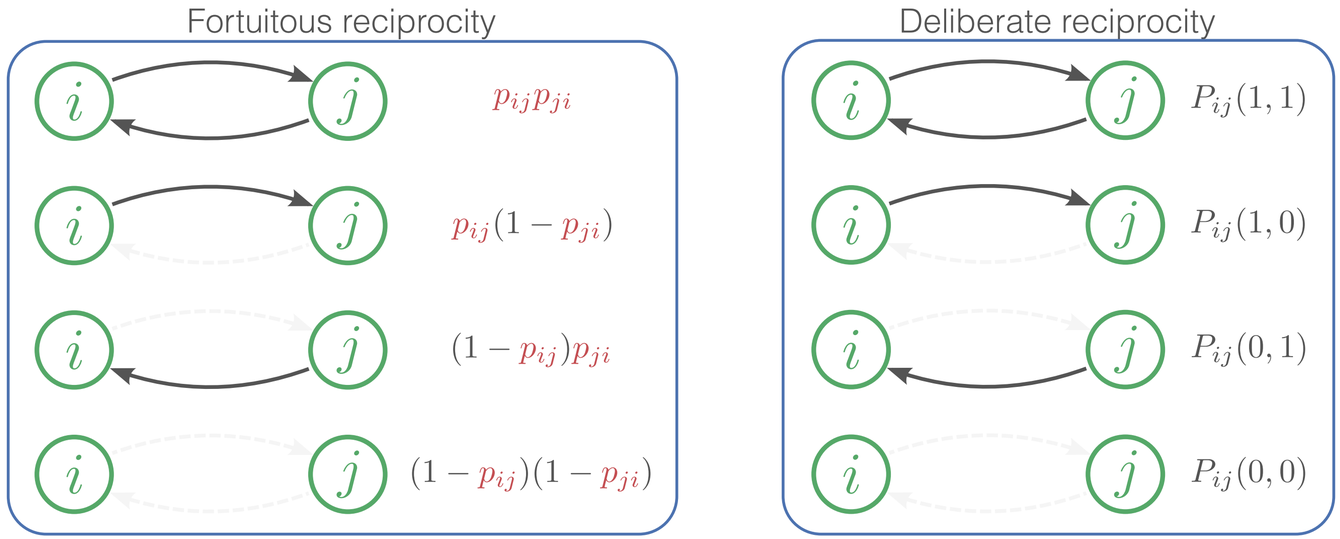
The directed-reciprocal $\mathbb{S}^1$ model: A new link between connections and distances

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j.



The directed-reciprocal $\mathbb{S}^1$ model: A new link between connections and distances

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j.



Level of reciprocity controlled with parameter $-1 \leq \nu \leq 1$

$$P_{ij}(1,1) = \begin{cases} (1-\nu)p_{ij}p_{ji} + \nu \min\{p_{ij}, p_{ji}\} & 0 \leq \nu \leq 1 \\ (1+\nu)p_{ij}p_{ji} + \nu(1-p_{ij}-p_{ji})H(p_{ij}+p_{ji}-1) & -1 \leq \nu \leq 0 \end{cases}$$

$\nu = 1$: maximal reciprocity

$\nu = 0$: fortuitous reciprocity

$\nu = -1$: minimal reciprocity

Condition 1: Preserves marginal probabilities

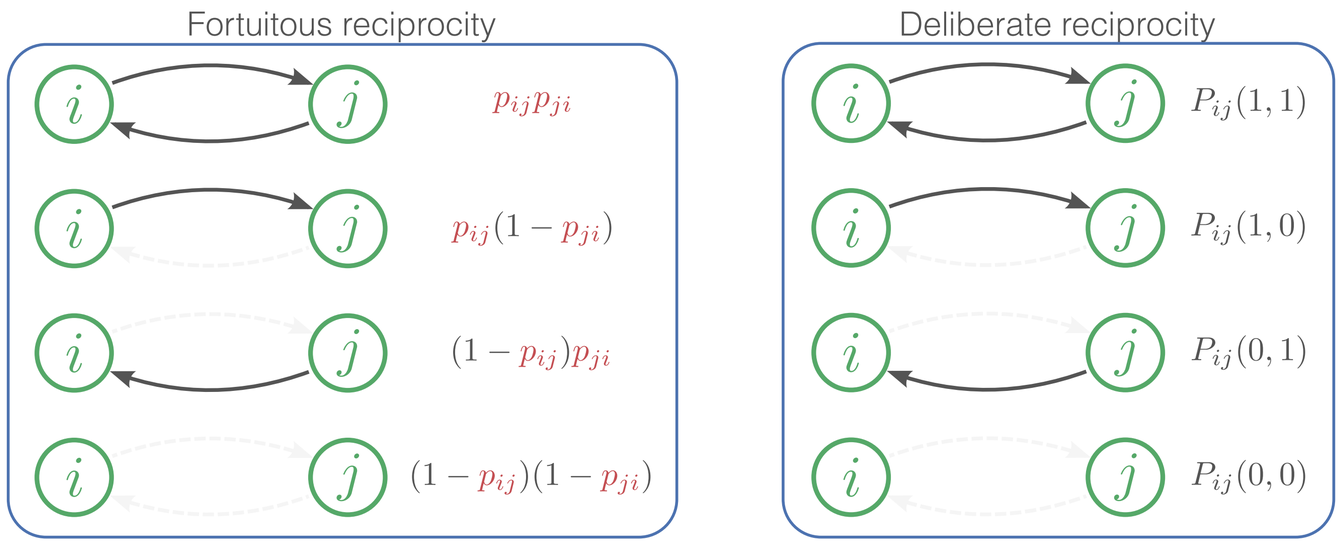
$$P_{ij}(1,0) + P_{ij}(1,1) = p_{ij}$$
$$P_{ij}(0,1) + P_{ij}(1,1) = p_{ji}$$

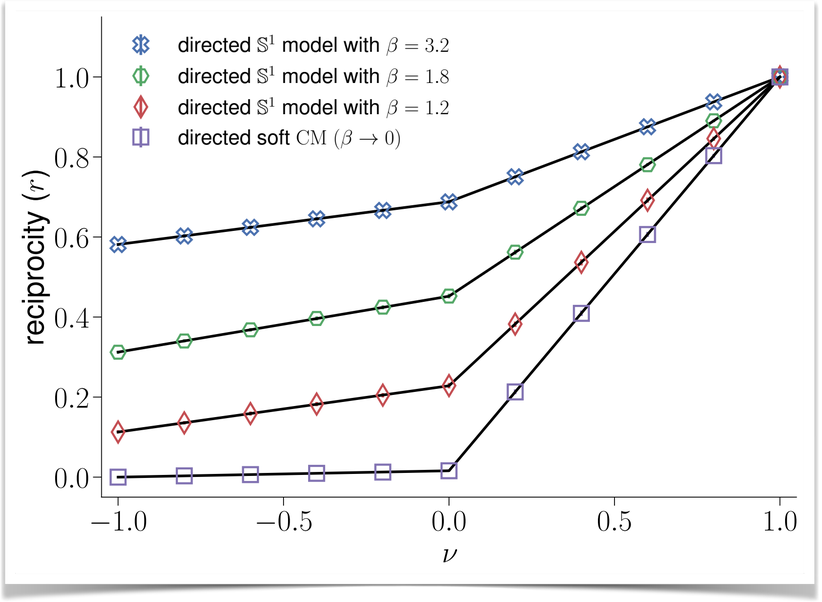
Condition 2: Normalized

$$\sum_{a_{ij}=0}^{1} \sum_{a_{ji}=0}^{1} P_{ij}(a_{ij}, a_{ji}) = 1$$

The directed-reciprocal $\mathbb{S}^1$ model: A new link between connections and distances

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j.





Level of reciprocity controlled with parameter $-1 \leq \nu \leq 1$

$$P_{ij}(1,1) = \begin{cases} (1-\nu)p_{ij}p_{ji} + \nu \min\{p_{ij}, p_{ji}\} & 0 \leq \nu \leq 1 \\ (1+\nu)p_{ij}p_{ji} + \nu(1-p_{ij}-p_{ji})H(p_{ij}+p_{ji}-1) & -1 \leq \nu \leq 0 \end{cases}$$

$\nu = 1$: maximal reciprocity

$\nu = 0$: fortuitous reciprocity

$\nu = -1$: minimal reciprocity

Condition 1: Preserves marginal probabilities

$$P_{ij}(1,0) + P_{ij}(1,1) = p_{ij}$$
$$P_{ij}(0,1) + P_{ij}(1,1) = p_{ji}$$

Condition 2: Normalized

$$\sum_{a_{ij}=0}^{1} \sum_{a_{ji}=0}^{1} P_{ij}(a_{ij}, a_{ji}) = 1$$

Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks

The likelihood of a network in the directed-reciprocal S^1 model is

$$\mathbb{P}\Big(\overbrace{\mathbb{A}}^{\text{adj. mat.}} \;\Big|\; \underbrace{\{\kappa_i^{\text{in}}\},\{\kappa_i^{\text{out}}\},\{\theta_i\}}_{\text{coordinates}}, \overbrace{\beta,\nu}^{\text{param.}} \Big) = \underbrace{\prod_{i=1}^{N}\prod_{j=i+1}^{N} P_{ij}(a_{ij},a_{ji})}_{\text{likelihood}}$$

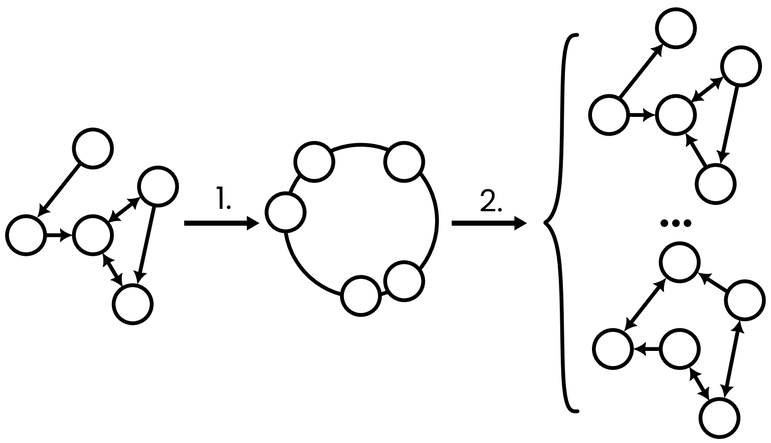
which we maximize using a combination of analytical results and numerical heuristics.

Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks

The likelihood of a network in the directed-reciprocal S^1 model is

$$\mathbb{P}\Big(\overbrace{\mathbb{A}}^{\text{adj. mat.}} \;\Big|\; \underbrace{\{\kappa_i^{\text{in}}\}, \{\kappa_i^{\text{out}}\}, \{\theta_i\}}_{\text{coordinates}}, \overbrace{\beta, \nu}^{\text{param.}} \Big) = \underbrace{\prod_{i=1}^{N} \prod_{j=i+1}^{N} P_{ij}(a_{ij}, a_{ji})}_{\text{likelihood}}$$

which we maximize using a combination of analytical results and numerical heuristics.

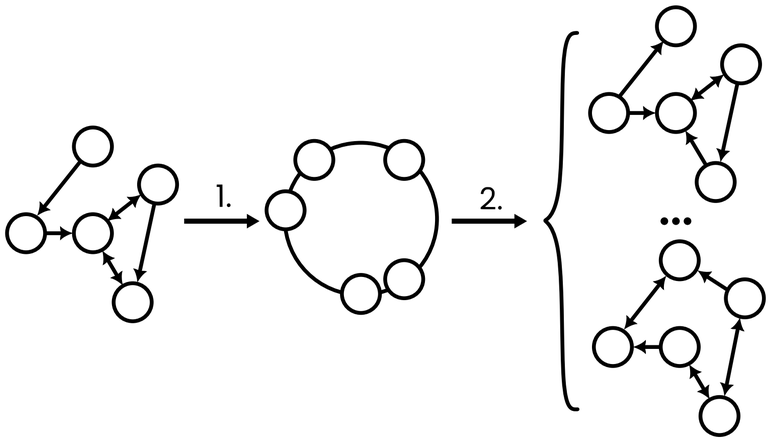


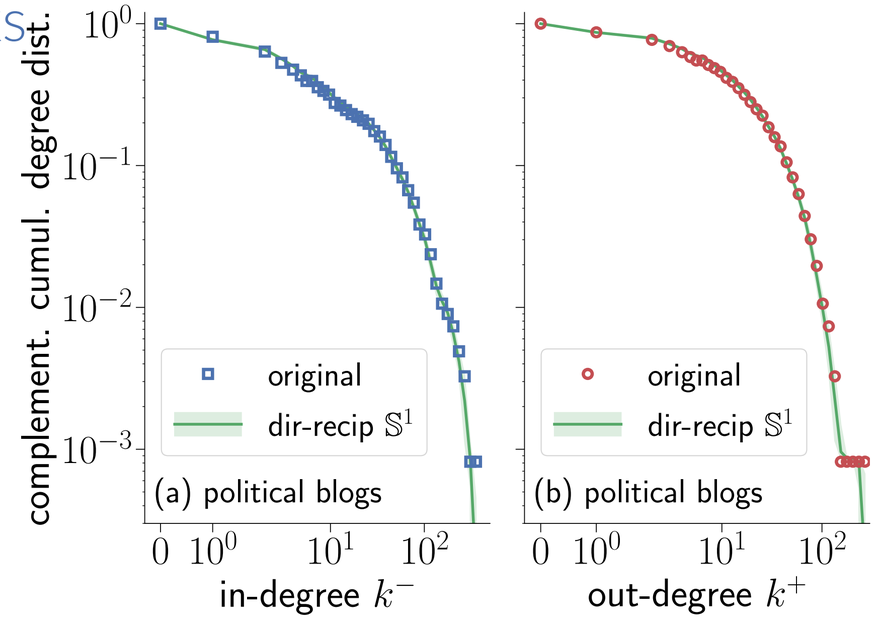
Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks

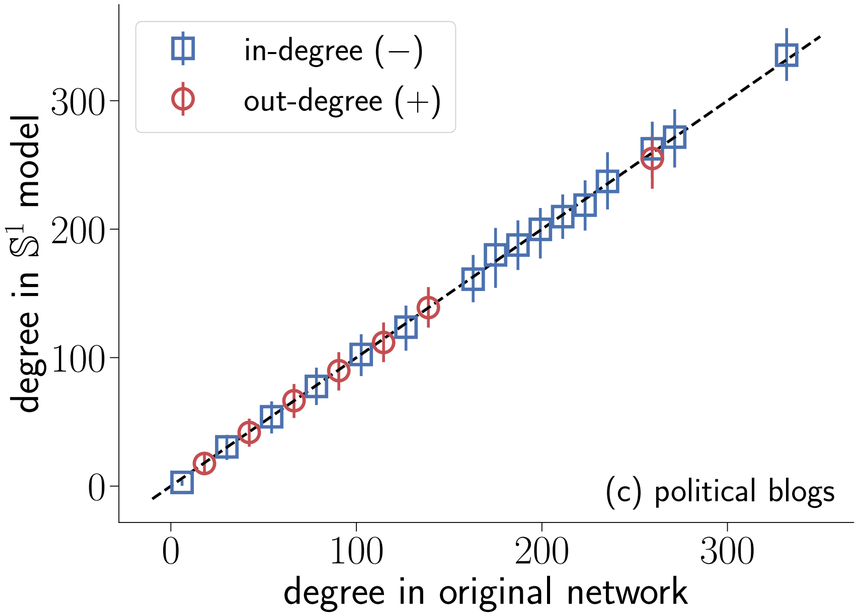
The likelihood of a network in the directed-reciprocal S^1 model is

$$\mathbb{P}\Big(\overbrace{\mathbb{A}}^{\text{adj. mat.}} \;\Big|\; \underbrace{\{\kappa_i^{\text{in}}\},\{\kappa_i^{\text{out}}\},\{\theta_i\}}_{\text{coordinates}}, \overbrace{\beta,\nu}^{\text{param.}} \Big) = \underbrace{\prod_{i=1}^{N}\prod_{j=i+1}^{N} P_{ij}(a_{ij},a_{ji})}_{\text{likelihood}}$$

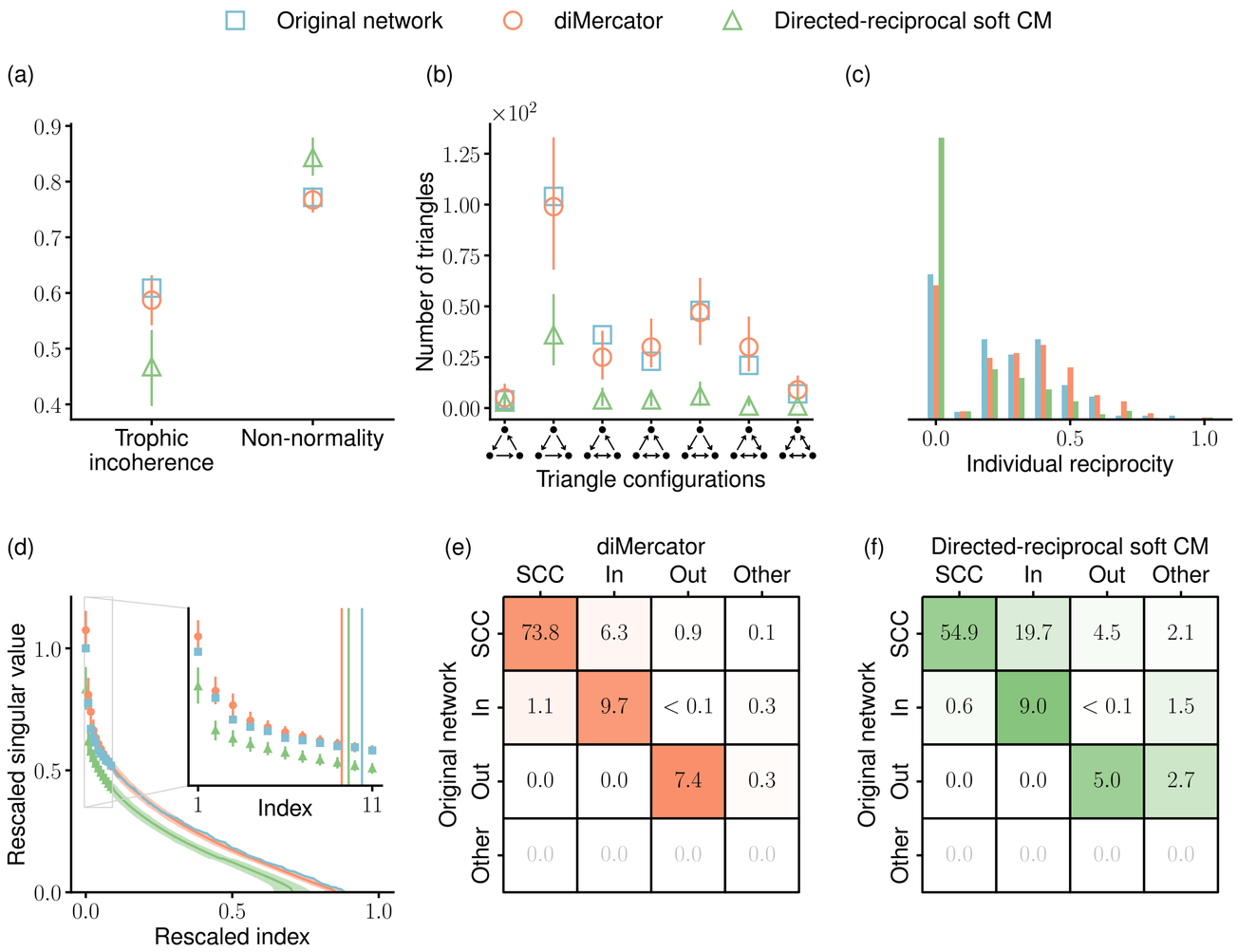
which we maximize using a combination of analytical results and numerical heuristics.



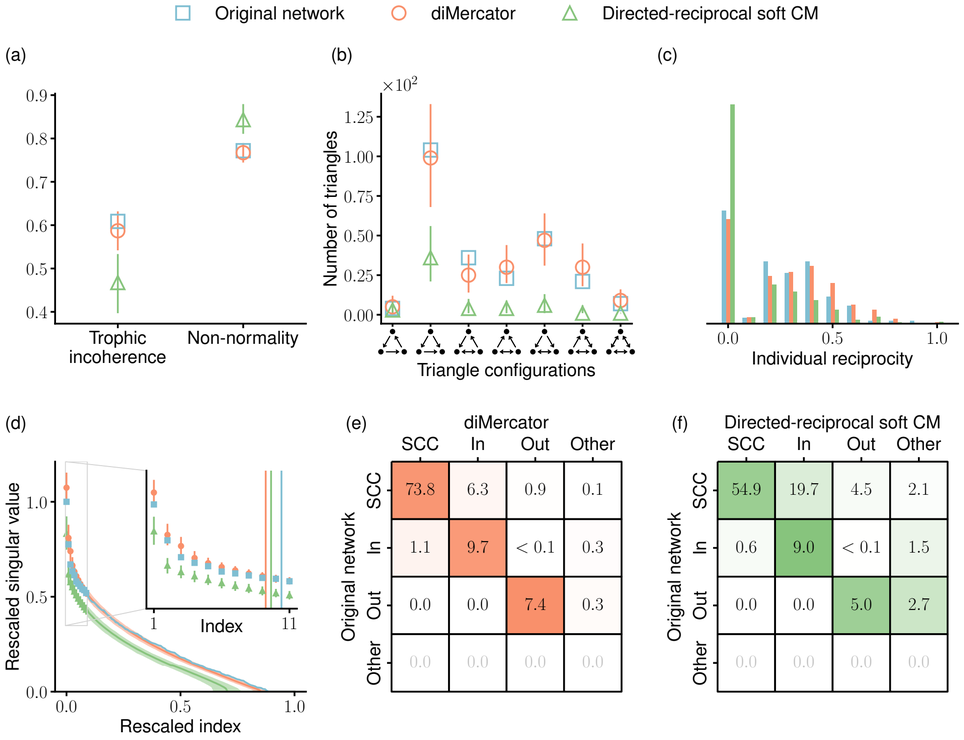




Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks

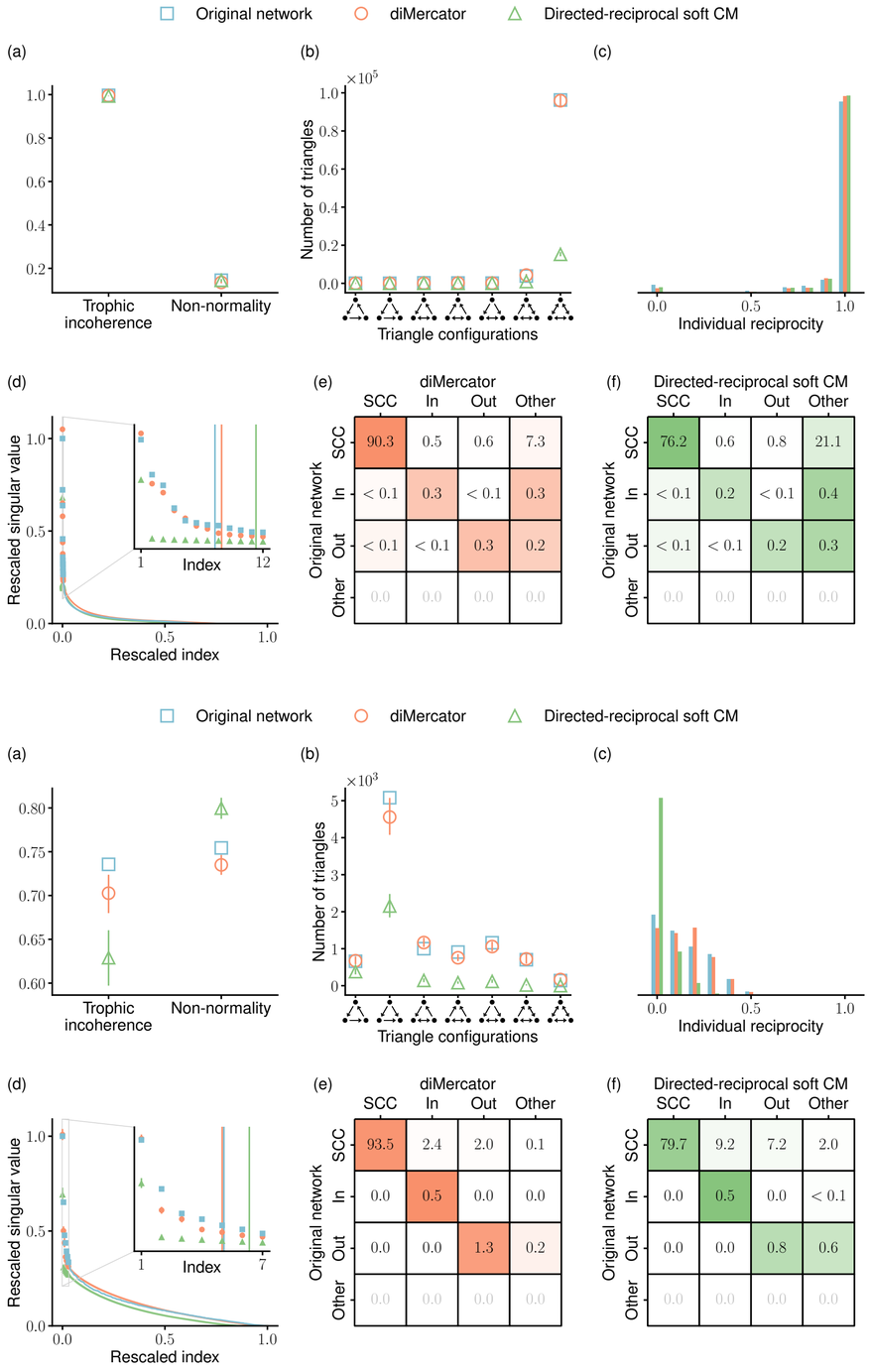


Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks



Fixing the degree sequence (on average), the global density of triangles and global reciprocity strongly constrains

- the non-normality [1]
- the trophic coherence [2]
- the clustering patterns
- the individual reciprocities
- the singular values
- the bow-tie organization



[1] Sci. Adv. 4, eaau9403 (2018)
[2] PNAS 111, 17923 (2014)

Main takeaways

- ▷ Rethinking the link between connections and distances allows the extension of the network geometry framework to directed networks.
- ▷ Assuming an underlying geometry is a simple modeling approach that can reproduce several complex topological properties observed in real networks.

Geometric description of clustering in directed networks

Antoine Allard ✉, M. Ángeles Serrano & Marián Boguñá

Nature Physics **20**, 150–156 (2024)

Local topological patterns and geometry parsimoniously explains the mesoscale organization of empirical complex networks

Jérémi Lesage,[1,2] M. Ángeles Serrano,[3,4,5] Marián Boguñá,[3,4] and Antoine Allard[1,2]

Dynamica

UNIVERSITÉ LAVAL

CIMMUL
Centre Interdisciplinaire en Modélisation Mathématique de l'Université Laval

Alliance de recherche numérique du Canada

NSERC CRSNG

Fonds de recherche Québec


J .Lesage


S. Lizotte


M. Á. Serrano


M. Boguñá

Slides

Main takeaways

- ▷ Rethinking the link between connections and distances allows the extension of the network geometry framework to directed networks.
- ▷ Assuming an underlying geometry is a simple modeling approach that can reproduce several complex topological properties observed in real networks.

Geometric description of clustering in directed networks

Antoine Allard ✉, M. Ángeles Serrano & Marián Boguñá

Nature Physics **20**, 150–156 (2024)

Local topological patterns and geometry parsimoniously explains the mesoscale organization of empirical complex networks

Jérémi Lesage,[1,2] M. Ángeles Serrano,[3,4,5] Marián Boguñá,[3,4] and Antoine Allard[1,2]

Dynamica

UNIVERSITÉ LAVAL

CIMMUL
Centre Interdisciplinaire en Modélisation Mathématique de l'Université Laval

Alliance de recherche numérique du Canada

NSERC CRSNG

Fonds de recherche Québec

npj | complexity

Editorial | Open access | Published: 17 April 2024

The path of complexity

Laurent Hébert-Dufresne ✉, Antoine Allard, Joshua Garland, Elizabeth A. Hobson & Luis Zaman

npj Complexity **1**, Article number: 4 (2024)

Aims & Scopes

npj Complexity considers research that explores all aspects of complex systems at the interface of multiple fields.

The journal covers a broad range of topics including but not limited to:

- Network science, including the analysis of complex networks such as social networks, biological networks, infrastructure and information networks
- Emergent phenomena, self-organization, and scaling in complex systems
- Nonlinear dynamics, chaos theory, and dynamical systems theory
- Agent-based modeling and simulation techniques for studying complex systems
- Complex adaptive systems, such as adaptive behavior in biological, ecological, and social systems
- Multi-scale analysis techniques across different levels of organization, from micro to macro scales
- Robustness and resilience of complex systems
- Evolutionary dynamics and game theory
- Digital humanities, dynamics of information, and science of science
- Information processing and computation in complex systems and artificial intelligence
- Interdisciplinary approaches to complex problems such as epidemics, climate change, misinformation, governance, and sustainability


J .Lesage


S. Lizotte


M. Á. Serrano


M. Boguñá

Slides